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Magneto hydrodynamic Fluid Flow past Contracting Surface Taking Account of Hall Current

Samuel K. Muondwe, Mathew Kinyanjui, David Theuri, Kang'ethe Giterere

Abstract: ***An unsteady, laminar hydrodynamic flow of an incompressible, viscous and electrically conducting Newtonian fluid over a porous contracting sheet in a rotating system considering hall current effects and heat source has been studied. The governing system of partial differential equations is transformed to dimensionless equations. The dimensionless equations are then solved numerically using the finite difference method. Using graphs the effects of various parameters on velocity, temperature and concentration are discussed The results obtained here are useful in applications on wire and glass fiber drawing in cooling of nuclear reactors, cooling of metallic plates and metallurgy.***

Index Terms**— MHD, Rotation, Hall current, contracting surfaces.**

I. INTRODUCTION

Fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. Fluids are classified as liquids and gases. Fluids can also be categorized as Newtonian and non-Newtonian fluids. For Newtonian fluids the shearing stress is linearly related to the rate of shearing strain but for non-Newtonian fluids shearing stress is not linearly related to the rate of shearing strain. Water, air, mercury, kerosene and thin lubricating oils are Newtonian fluids whereas paints, coal tar, blood and grease are non-Newtonian fluids. Investigation of hydro magnetic natural convection flow with heat and mass transfer in porous media has been studied by several researchers due to its applications in geophysics, astrophysics, aeronautics, meteorology, electronics, chemical, and metallurgy and petroleum industries. [1] studied free convective mass transfer flow past infinite vertical porous plate with variable suction and Soret effect and concluded that increase in magnetic intensity contribute to the decrease in the velocity. [2] Analyzed radiation effects on MHD free convection flow of Kuvshinshiki fluid with mass transfer past a vertical porous plate through a porous media. [2] Concluded that temperature decreases with increase in radiation parameter. Most researchers ignore Hall current while applying Ohm's law because it has no significant effect for small and average values of the magnetic field. The effects of Hall current are very important in the presence of a strong magnetic field, because for strong magnetic field electromagnetic force is prominent. [3] studied Hall effect on transient MHD flow past an impulsively started vertical plate in a porous medium with ramped temperature, rotation and heat absorption and concluded that for both ramped temperature and isothermal plates the primary and secondary motions are accelerated due Hall current, further they noted that skin friction (drag force due to primary velocity) rises with increasing values of magnetic parameter for ramped temperature. [4] Studied effect of Hall current, radiation and rotation on natural convection heat and mass transfer flow past a moving vertical plate. [4] Concluded that an increase in rotation parameter causes a decrease in primary velocity, secondary velocity in the region away from the plate and reverse effect is observed on secondary velocity in a region near the plate. [5] Studied unsteady hydro-magnetic couette flow of a viscous, incompressible and electrically conducting fluid between two infinitely long parallel porous plates, taking Hall current into account, in the presence of a transverse magnetic field. Fluid flow within the channel is induced due to impulsive movement of the lower plate of the channel Uniform magnetic fields is in the direction orthogonal to the permeable plates, uniform suction and injection through the plates are applied. [5] Concluded that when the magnetic field is considered to be fixed relative to the plate, the flow is accelerated by the magnetic field. However, the magnetic field retards the fluid flow when the magnetic field is fixed relative to the flow. [6] Studied Hall effect on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with heat source/sink. [6] Concluded that skin friction decreases for Hartman number less than critical value, afterwards it starts to increase.

[7] Studied effects of Hall current, rotation and Soret effects on MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium. [7] concluded that; Hall current tends to accelerate secondary fluid velocity throughout the boundary layer region whereas it has a reverse effect on the primary fluid velocity throughout the boundary layer region; Rotation tends to accelerate secondary fluid velocity throughout the boundary layer region whereas it has a reverse effect on the primary fluid velocity throughout the boundary layer region; [8] analyzed effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction, and concluded that increase in mass suction parameter it causes momentum boundary layer thickness thinner. [9] Studied fluid flow and heat transfer of Nano fluid containing motile gyrotactic micro-organism passing a nonlinear stressing vertical sheet in the presence of non- uniform magnetic field. [10] studied Combined effects of variable magnetic field and porous medium on the flow of MHD fluid due to exponentially shrinking sheet and concluded that the suction parameter and porous medium permeability increase the velocity profiles, whereas, magnetic field has reverse effect, the Prandtl number and suction parameter have a reducing effect on the temperature profiles. The aim of this work is therefore to study MHD stokes fluid flow problem and combined effects of various parameters on velocity, temperature and concentration past a porous contracting surface in a rotating system taking account of Hall current.

II. MATHEMATICAL FORMULATIONS

Consider an unsteady MHD laminar boundary layer flow of an incompressible, electrically conducting, and viscous Newtonian fluid past a contracting electrically non-conducting sheet embedded in porous media with heat and mass transfer in a rotating system. The contracting sheet is permeable to allow for a possible suction and is continuously contracting in the x-axis direction in the plane with a velocity $v = -cx^n$, where c is negative for contacting plate and n is nonlinear contracting parameter. At distance L units away, a second impermeable and electrically non conducting sheet is placed parallel to the contracting sheet for closed flow. The system is rotated with constant angular velocity Ω; the y axis is taken to be infinite. The pressure gradient is in the positive x-axis direction, and is directed upward parallel to the direction of gravity. Additionally, a variable magnetic field is applied normal to the two plates.

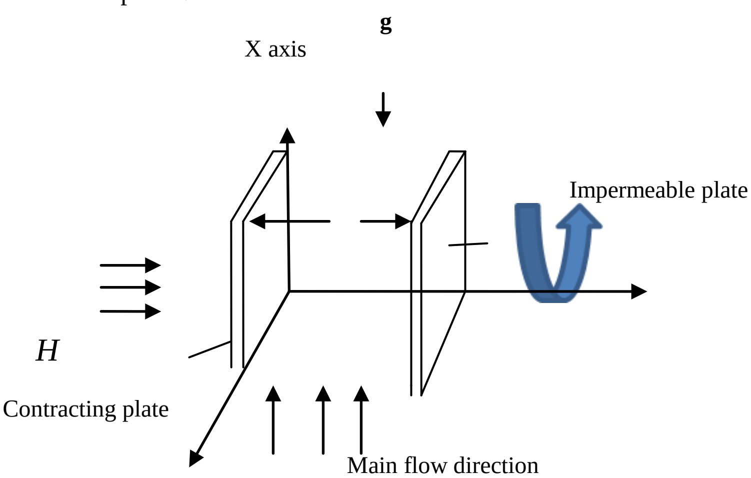

Fig 1: Flow configuration

If Q is the temperature dependent volumetric heat generation parameter, then $Q > 0$ represents a heat source, and $Q < 0$ represents a heat sink. Taking Hall current into account, the generalized Ohm's law can be written as

$$J + \frac{\omega_e \tau_e}{H_0} J + H = \sigma\left(E + \mu_e q \times H\right) \qquad (1)$$

In equation (1) the electron pressure gradient (for weakly ionized fluid), ion-slip and thermo-electric effects are neglected. Under these assumptions, and in the absence of electric field, equation (1) becomes

$$J_x + mJ_y = \sigma \mu_e H_0 v \qquad (2)$$

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$$J_y - mJ_x = -\sigma\mu_e H_0 u \tag{3}$$

Where $m = \omega_e \tau_e$ is the Hall parameter? Solving equations (2) and (3) for and yields

$$J_x = \frac{\sigma\mu_e H_0}{1+m^2}(v + mu) \tag{4}$$

$$J_y = \frac{\sigma\mu_e H_0}{1+m^2}(mv - u) \tag{5}$$

The Lorentz force $J \times B$ yields

$$J_y B_0 i - J_x B_0 j \tag{6}$$

If Q is the temperature dependent volumetric heat generation parameter, then $Q > 0$ represents a heat source, and $Q < 0$ represents a heat sink. Incorporating equations (4) to (6) into the equations of momentum e energy and concentration, the following respective equations of momentum, energy and concentration are obtained

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \omega_0 \frac{\partial u}{\partial z} - 2\Omega v = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - u\frac{\nu}{k} + \frac{\sigma\mu^2 H^2}{\rho\left(1+m^2\right)}(mv - u) \tag{7}$$

$$+\beta g\left(T - T_\infty\right) + \beta^* g\left(C - C_\infty\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} - \omega_0 \frac{\partial v}{\partial z} + 2\Omega u = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) - v\frac{\nu}{k} - \frac{\sigma\mu^2 H^2}{\rho\left(1+m^2\right)}(v + mu) \tag{8}$$

Equation of energy:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} - \omega_0 \frac{\partial T}{\partial z} = \frac{k_f}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{D_m k_T}{c_s c_p}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right)$$

$$+\frac{\sigma}{\rho c_p}\mu^2 H_0^2\left(u^2 + v^2\right) + \frac{Q_0}{\rho c_p}(T - T_\infty) + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \tag{9}$$

$$-\frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial z^2}$$

Equation of concentration:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} - \omega_0 \frac{\partial C}{\partial z} = D_m\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) + \frac{D_m k_T}{T}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) - u\frac{\partial C}{\partial x} + \omega_0 \frac{\partial C}{\partial z} \tag{10}$$

The initial and boundary conditions:

$t \le 0 : u = 0, v = 0, \omega = 0, T = 0, H = 0, C = 0$ at $0 \le z \le L$

$t > 0 : u = u_\infty, v = 0, T = T_\infty, C = C_\infty, H = H_0, at, x = 0$ Channel entrance

$t > 0 : u = -cx^n, v = 0, T = T_w, C = C_w, H = H_0 x^{\frac{n-1}{2}}, at, z = 0$ (porous wall)

$t > 0 : u = 0, v = 0, T = T_w, C = C_w, H = H_0, at, z = L$ (impermeable wall) (11)

The non-dimensional Variables are defined as follows'

$$u' = \frac{u}{U_\infty},\ v' = \frac{v}{U_\infty},\ \omega' = \frac{\omega_0}{U_\infty},\ t' = \frac{U\infty t}{L},\ z' = \frac{z}{L},\ x' = \frac{x}{L},\ T' = \frac{T - T\infty}{T_w - T_\infty},\ H = H_0 H' \tag{12}$$

Governing equations in non-dimensional form

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$$\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}-\omega'\frac{\partial u'}{\partial z'}-R_0 rv'=\frac{1}{R_e}\left(\frac{\partial^2 u'}{\partial x'^2}+\frac{\partial^2 u'}{\partial z'^2}\right)-Xiu'-Mu'+\frac{m}{1+m^2}\left(mv'-u'\right)$$
$$+Gr_\theta T'+Gr_c C' \tag{13}$$

$$\frac{\partial v'}{\partial t'}+v'\frac{\partial v'}{\partial x'}-\omega'\frac{\partial v'}{\partial z'}+Eru'=\frac{1}{R_m}\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial z'^2}\right)-Xiu'-Mu'+\frac{m}{1+m^2}\left(v'+mu'\right) \tag{14}$$

Equation of energy

$$\frac{\partial T'}{\partial t'}+u'\frac{\partial T'}{\partial x'}-\omega_0\frac{\partial T'}{\partial z'}=\frac{1}{R_m}\left[\frac{1}{pr}\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial z'^2}\right)\right]+D_f R_m\left(\frac{\partial^2 C'}{\partial x'^2}+\frac{\partial^2 C'}{\partial z'^2}\right)+\frac{Ec}{R_m}\left[\left(\frac{\partial u'}{\partial z'}\right)^2+\left(\frac{\partial v'}{\partial z'}\right)^2\right]$$
$$+QT'-\frac{4}{3N\Pr R_m}\left(\frac{\partial^2 T'}{\partial z'^2}\right)+RR_m\left(u'^2+v'^2\right)$$

(15)

Equation of concentration

$$\frac{\partial C'}{\partial t'}+u'\frac{\partial C'}{\partial x'}-\omega_0\frac{\partial C'}{\partial z'}=\frac{1}{\operatorname{Re} Sc}\left(\frac{\partial^2 C'}{\partial x'^2}+\frac{\partial^2 C'}{\partial z'^2}\right)+\frac{Sr}{\operatorname{Re}}\left(\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial z'^2}\right) \tag{16}$$

Where $Q=\frac{Q_0 H}{\rho u c_p}$ is the non-dimensional heat source/sink parameter. Where $M=\frac{\sigma\mu_e^2 H^2}{\rho U_\infty}$ is the magnetic field parameter, $\operatorname{Re}=\frac{U_\infty L}{\vartheta}$ is the Reynolds number, $Xi=\frac{\vartheta L}{U_\infty k_p}$ is the permeability parameter, $Gr_\theta=\frac{\beta g L(T_w-T_\infty)}{U_\infty^2}$ is the local temperature Grashof number, $Ro=\frac{2\Omega}{U_\infty^2}$ is rotational parameter. Where $\Pr=\frac{\mu c_p}{k_f}$ is prandlt number, $N=\frac{k^* k_f}{4\sigma^* T_\infty^3}$ is the radiation parameter, $D_f=\frac{\rho D_m k_f(C_w-C_\infty)}{\mu c_s c_p(T_w-T_\infty)}$ is Dufour number , $Ec=\frac{U^2}{c_p\Delta T}$ is the Eckert number, $R=\frac{\sigma\mu_e^2 H^2}{\rho^2 c_p\Delta T}$ is the Joule heating parameter and ΔT represents the temperature differences. (T_w-T_∞).

The initial and boundary conditions in non-dimensional form:

$t'<0: u'=0, v'=0, T'=0, C'=0, H'=0, at, z=0$ at $0\le z\le L$

$t'>0: u'=1, v'=0, T'=1, C'=1, H'=1, at, z=0$ Channel entrance

$$t'>0: u'=\frac{-cLx^n}{U_\infty}, v'=0, T'=1, C'=1, H'=x^{\frac{n-1}{2}}, at, z=0 \text{ (Porous wall)} \tag{17}$$

$t'>0: u'=0, v'=0, T'=, C'=1, H'=1, at, z=L$ (Impermeable wall)

III. METHOD OF SOLUTIONS

The equations are solved using finite differences method and implemented in MATLAB. The finite difference method replaces each PDE with a discrete approximation for space and time domain.

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The Finite Difference form of the equation of momentum along the x-axis is

$$\frac{U_{i,j}^{k+1}-U_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\left(\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta x}\right)-w_0\left(\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta z}\right)-\frac{R_0}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)$$
$$=\frac{1}{2(\Delta x)^2\operatorname{Re}}\left(U_{i+1,j}^{k+1}-2U_{i,j}^{k+1}+U_{i-1,j}^{k+1}+U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}\right)+\frac{1}{2(\Delta z)^2\operatorname{Re}}\left(U_{i+1,j}^{k+1}-2U_{i,j}^{k+1}+U_{i-1,j}^{k+1}+U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}\right)$$
$$-\frac{Xi}{2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)-\frac{M}{2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)+\frac{Gr_\theta}{2}\left(T_{i,j}^{k+1}+T_{i,j}^{k}\right)+\frac{Gr_c}{2}\left(C_{i,j}^{k+1}+C_{i,j}^{k}\right)+\frac{m}{1+m^2}\left[\frac{m}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)-\frac{1}{2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)\right] \quad (18)$$

Making $U_{i,j}^{k+1}$ the subject of the formula

$$U_{i,j}^{k+1}=U_{i,j}^{k}+\frac{\Delta t}{2(\Delta x)^2\operatorname{Re}}\left(U_{i+1,j}^{k+1}+U_{i-1,j}^{k+1}+U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}\right)$$
$$+\frac{\Delta t}{2(\Delta z)^2\operatorname{Re}}\left(U_{i+1,j}^{k+1}+U_{i-1,j}^{k+1}+U_{i+1,j}^{k}-2U_{i,j}^{k}+U_{i-1,j}^{k}\right)+\frac{\Delta t}{2\Delta x}U_{i,j}^{k}\left(U_{i-1,j}^{k+1}-U_{i,j}^{k}+U_{i-1,j}^{k}\right)$$
$$+\frac{\Delta t w_0}{2\Delta z}\left(U_{i-1,j}^{k+1}-U_{i,j}^{k}+U_{i-1,j}^{k}\right)-\frac{\Delta tXi}{2}U_{i,j}^{k}-\frac{\Delta tM}{2}U_{i,j}^{k}+\frac{\Delta tGr_\theta}{2}\left(T_{i,j}^{k+1}+T_{i,j}^{k}\right)+\frac{\Delta tGr_c}{2}\left(C_{i,j}^{k+1}+C_{i,j}^{k}\right)$$
$$+\frac{\Delta tR_0}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)+\frac{m}{2+2m^2}\left[\frac{m}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)-U_{i,j}^{k}\right]/(1+\frac{\Delta t}{2\Delta x}U_{i,j}^{k}-\frac{w_0\Delta t}{2\Delta z}+\frac{\Delta t}{\operatorname{Re}(\Delta z)^2}+\frac{\Delta t}{\operatorname{Re}(\Delta x)^2}$$
$$+\frac{\Delta tXi}{2}+\frac{\Delta tM}{2}+\frac{1}{2}\left(\frac{m\Delta t}{1+m^2}\right)) \quad (19)$$

Momentum equation along y-axis

$$\frac{V_{i,j}^{k+1}-V_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\left(\frac{V_{i,j}^{k+1}-V_{i-1,j}^{k}+V_{i,j}^{k}-V_{i-1,j}^{k}}{2\Delta x}\right)-w_0\left(\frac{V_{i,j}^{k+1}-V_{i-1,j}^{k}+V_{i,j}^{k}-V_{i-1,j}^{k}}{2\Delta z}\right)$$
$$+\frac{R_0}{2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)=\frac{1}{2(\Delta x)^2\operatorname{Re}}\left(V_{i+1,j}^{k+1}-2V_{i,j}^{k+1}+V_{i-1,j}^{k+1}+V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}\right) \quad (20)$$
$$+\frac{1}{2(\Delta z)^2\operatorname{Re}}\left(V_{i+1,j}^{k+1}-2V_{i,j}^{k+1}+V_{i-1,j}^{k+1}+V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}\right)-\frac{Xi}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)-\frac{M}{2}\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)$$
$$+\frac{m}{2+2m^2}\left[\left(V_{i,j}^{k+1}+V_{i,j}^{k}\right)+m\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)\right]$$

Making $V_{i,j}^{k+1}$ the subject of the formula

$$V_{i,j}^{k+1}=V_{i,j}^{k}+\frac{\Delta t}{2(\Delta x)^2\operatorname{Re}}\left(V_{i+1,j}^{k+1}+V_{i-1,j}^{k+1}+V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}\right)+\frac{\Delta t}{2(\Delta z)^2\operatorname{Re}}\left(V_{i+1,j}^{k+1}+V_{i-1,j}^{k+1}+V_{i+1,j}^{k}-2V_{i,j}^{k}+V_{i-1,j}^{k}\right)$$
$$+\frac{\Delta t}{2\Delta x}U_{i,j}^{k}\left(V_{i-1,j}^{k+1}-V_{i,j}^{k}+V_{i-1,j}^{k}\right)+\frac{\Delta t w_0}{2\Delta z}\left(V_{i-1,j}^{k+1}-V_{i,j}^{k}+V_{i-1,j}^{k}\right)-\frac{\Delta tXi}{2}V_{i,j}^{k}-\frac{\Delta tM}{2}V_{i,j}^{k}$$
$$+\frac{\Delta tR_0}{2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)+\frac{m\Delta t}{2+2m^2}V_{i,j}^{k}+\frac{m^2}{2+2m^2}\left(U_{i,j}^{k+1}+U_{i,j}^{k}\right)/(1+\frac{\Delta t}{2\Delta x}V_{i,j}^{k}-\frac{w_0\Delta t}{2\Delta z}+\frac{\Delta t}{\operatorname{Re}(\Delta z)^2}$$
$$+\frac{\Delta t}{\operatorname{Re}(\Delta x)^2}+\frac{\Delta tXi}{2}+\frac{\Delta tM}{2}-\frac{m\Delta t}{2+2m^2}) \quad (21)$$

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Energy equation in finite difference form

$$\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\left(\frac{T_{i,j}^{k+1}-T_{i-1,j}^{k}+T_{i,j}^{k}-T_{i-1,j}^{k}}{2\Delta x}\right)-w_0\left(\frac{T_{i,j}^{k+1}-T_{i-1,j}^{k}+T_{i,j}^{k}-T_{i-1,j}^{k}}{2\Delta z}\right)$$

$$=\frac{1}{2(\Delta x)^2\Pr\mathrm{Re}}\left(T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)$$

$$+\frac{1}{2(\Delta z)^2\Pr\mathrm{Re}}\left(T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)+\frac{Ec}{\mathrm{Re}}\left[\left(\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta x}\right)^2+\left(\frac{V_{i,j}^{k+1}-V_{i-1,j}^{k+1}+V_{i,j}^{k}-V_{i-1,j}^{k}}{2\Delta z}\right)^2\right]$$

$$-\frac{2}{3Ni\Pr\mathrm{Re}\left(\Delta z\right)^2}\left(T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)+R\,\mathrm{Re}\left[\left(\frac{U_{i,j}^{k+1}+U_{i,j}^{k}}{2}\right)^2+\left(\frac{V_{i,j}^{k+1}+V_{i,j}^{k}}{2}\right)^2\right]$$

$$+D_f\,\mathrm{Re}\left(\frac{C_{i+1,j}^{k+1}-2C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}}{2\left(\Delta x\right)^2}\right)+D_f\,\mathrm{Re}\left(\frac{C_{i+1,j}^{k+1}-2C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}}{2\left(\Delta z\right)^2}\right)+\frac{Q}{2}\left[T_{i,j}^{k+1}+T_{i,j}^{k}\right]$$

(22)

Making $T_{i,j}^{k+1}$ the subject of the formula

$$T_{i,j}^{k+1}=T_{i,j}^{k}+\frac{\Delta t}{2\left(\Delta x\right)^2\Pr\mathrm{Re}}\left(T_{i+1,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)$$

$$+\frac{\Delta t}{2\left(\Delta z\right)^2\Pr\mathrm{Re}}\left(T_{i+1,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)+\frac{\Delta t}{2\Delta x}U_{i,j}^{k}\left(T_{i-1,j}^{k+1}-T_{i,j}^{k}+T_{i-1,j}^{k}\right)$$

$$+\frac{\Delta t w_0}{2\Delta z}\left(T_{i-1,j}^{k+1}-T_{i,j}^{k}+T_{i-1,j}^{k}\right)-\frac{4\Delta t}{2\left(\Delta z\right)^2 Ni\Pr\mathrm{Re}}\left(T_{i+1,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}\right)$$

$$+\frac{\Delta tEc}{\mathrm{Re}}\left[\left(\frac{U_{i,j}^{k+1}-U_{i-1,j}^{k+1}+U_{i,j}^{k}-U_{i-1,j}^{k}}{2\Delta x}\right)^2+\left(\frac{V_{i,j}^{k+1}-V_{i-1,j}^{k+1}+V_{i,j}^{k}-V_{i-1,j}^{k}}{2\Delta z}\right)^2\right]+\Delta tR\,\mathrm{Re}\left[\left(\frac{U_{i,j}^{k+1}+U_{i,j}^{k}}{2}\right)^2+\left(\frac{V_{i,j}^{k+1}+V_{i,j}^{k}}{2}\right)^2\right]$$

$$+\Delta tD_f\,\mathrm{Re}\left[\left(\frac{C_{i+1,j}^{k+1}+C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}}{2\left(\Delta x\right)^2}\right)+\left(\frac{C_{i+1,j}^{k+1}+C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}}{2\left(\Delta x\right)^2}\right)\right]$$

$$+\frac{Q\Delta t}{2}T_{i,j}^{k}\Big/\left[1+\frac{\Delta t}{2\Delta x}U_{i,j}^{k}-\frac{w_0\Delta t}{2\Delta z}+\frac{\Delta t}{\Pr\mathrm{Re}(\Delta z)^2}+\frac{\Delta t}{\Pr\mathrm{Re}(\Delta x)^2}+\frac{4\Delta t}{3\left(\Delta z\right)^2 Ni\Pr\mathrm{Re}}-\frac{Q\Delta t}{2}\right]$$

(23)

Concentration equation in finite difference form

$$\frac{C_{i,j}^{k+1}-C_{i,j}^{k}}{\Delta t}+U_{i,j}^{k}\left(\frac{C_{i,j}^{k+1}-C_{i-1,j}^{k}+C_{i,j}^{k}-C_{i-1,j}^{k}}{2\Delta x}\right)-w_0\left(\frac{C_{i,j}^{k+1}-C_{i-1,j}^{k}+C_{i,j}^{k}-C_{i-1,j}^{k}}{2\Delta z}\right)$$

$$=\frac{1}{2(\Delta x)^2 Sc\,\mathrm{Re}}\left(C_{i+1,j}^{k+1}-2C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}\right) \qquad (24)$$

$$+\frac{1}{2(\Delta z)^2 Sc\,\mathrm{Re}}\left(C_{i+1,j}^{k+1}-2C_{i,j}^{k+1}+C_{i-1,j}^{k+1}+C_{i+1,j}^{k}-2C_{i,j}^{k}+C_{i-1,j}^{k}\right)$$

$$+\frac{Sr}{\mathrm{Re}}\left[\left(\frac{T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}}{2\left(\Delta x\right)^2}\right)+\left(\frac{T_{i+1,j}^{k+1}-2T_{i,j}^{k+1}+T_{i-1,j}^{k+1}+T_{i+1,j}^{k}-2T_{i,j}^{k}+T_{i-1,j}^{k}}{2\left(\Delta z\right)^2}\right)\right]$$

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Making $C_{i,j}^{k+1}$ the subject of the formula

$$
\begin{aligned}
C_{i,j}^{k+1} &= C_{i,j}^{k} + \frac{\Delta t}{2(\Delta x)^2\, Sc\,\mathrm{Re}}\left(C_{i+1,j}^{k+1} + C_{i-1,j}^{k+1} + C_{i+1,j}^{k} - 2C_{i,j}^{k} + C_{i-1,j}^{k}\right) \\
&+ \frac{\Delta t}{2(\Delta z)^2\, Sc\,\mathrm{Re}}\left(C_{i+1,j}^{k+1} + C_{i-1,j}^{k+1} + C_{i+1,j}^{k} - 2C_{i,j}^{k} + C_{i-1,j}^{k}\right) + \frac{\Delta t}{2\Delta x}U_{i,j}^{k}\left(C_{i-1,j}^{k+1} - C_{i,j}^{k} + C_{i-1,j}^{k}\right) \\
&+ \frac{\Delta t w_0}{2\Delta z}\left(C_{i-1,j}^{k+1} - C_{i,j}^{k} + C_{i-1,j}^{k}\right) + \frac{\Delta t Sr}{2(\Delta x)^2\,\mathrm{Re}}\left(T_{i+1,j}^{k+1} - 2T_{i,j}^{k+1} + T_{i-1,j}^{k+1} + T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}\right) \\
&+ \frac{\Delta t Sr}{2(\Delta z)^2\,\mathrm{Re}}\left(T_{i+1,j}^{k+1} - 2T_{i,j}^{k+1} + T_{i-1,j}^{k+1} + T_{i+1,j}^{k} - 2T_{i,j}^{k} + T_{i-1,j}^{k}\right) \\
&/\left[1 + \frac{\Delta t}{2\Delta x}U_{i,j}^{k} - \frac{w_0 \Delta t}{2\Delta z} + \frac{\Delta t}{Sc\,\mathrm{Re}(\Delta z)^2} + \frac{\Delta t}{Sc\,\mathrm{Re}(\Delta x)^2}\right]
\end{aligned}
\tag{25}
$$

IV. RESULTS AND DISUSSION

In order to get a physical insight into the problem at hand, the velocity, temperature and concentration have been analyzed by assigning numerical values to various non-dimensional parameters. These parameters are input into a computer program where each parameter is varied at a time. The various parameters that have been varied include, Magnetic field M, Rotational parameter Ro, Permeability parameter Xi, , time t, , Radiation parameter, Prandtl number and suction parameter.

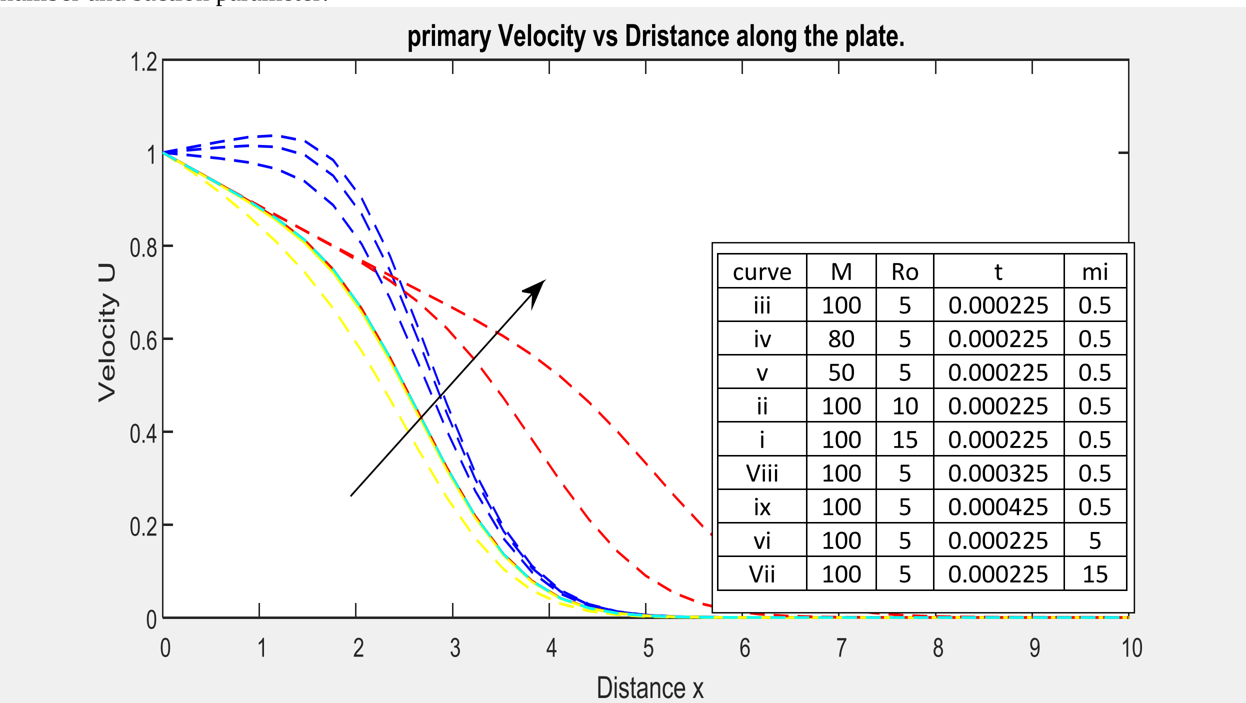

Fig 2: Primary velocity profiles for different values of Magnetic Parameter M, Rotation parameter Ro, time t and Hall parameter mi, for heat source and positive Grashof number.

From figure 2 and 3 it is noted that in the presence of a heat source $Q > 0$, An increase in Magnetic Parameter M leads to a decrease in the magnitude of primary and secondary velocity profiles. An increase in the Rotation parameter Ro leads to a decrease in the primary and an increase in secondary velocity profiles. This is because when the frictional layer at the moving plate is suddenly set into the rotation then the Coriolis force tends to oppose primary velocity by generating cross flow i.e. secondary velocity. An increase in time causes an increase in primary

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and secondary velocity profiles. An increase in Hall parameter mi, leads to a slight increase in the magnitude of primary velocity profiles and a decrease in secondary velocity profiles.

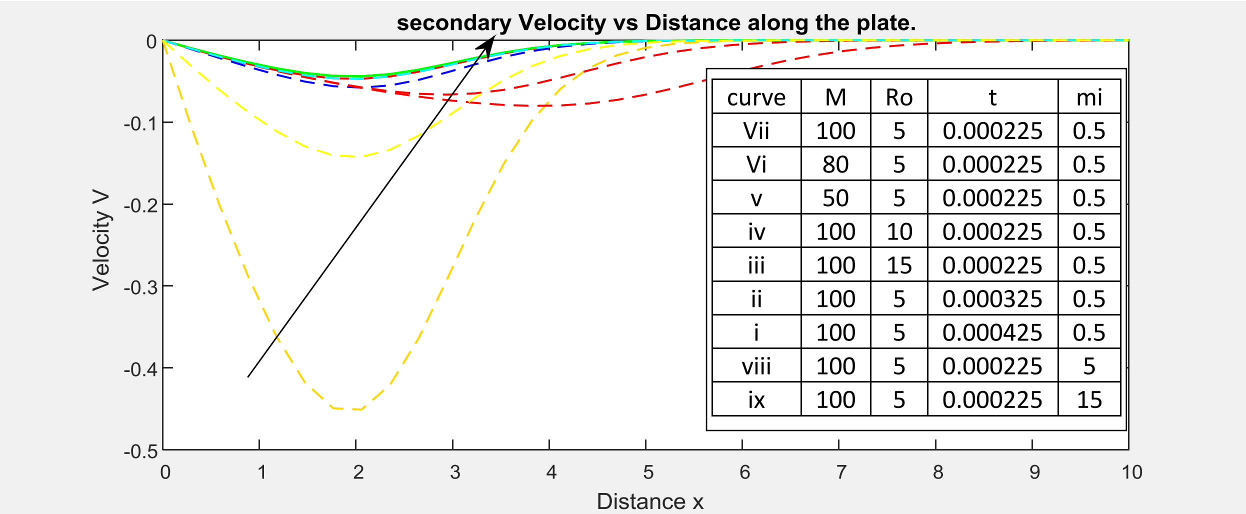

Fig 3: Secondary velocity profiles for different values of Magnetic Parameter M, Rotation parameter Ro, time t and Hall parameter mi, for heat source and positive Grashof number.

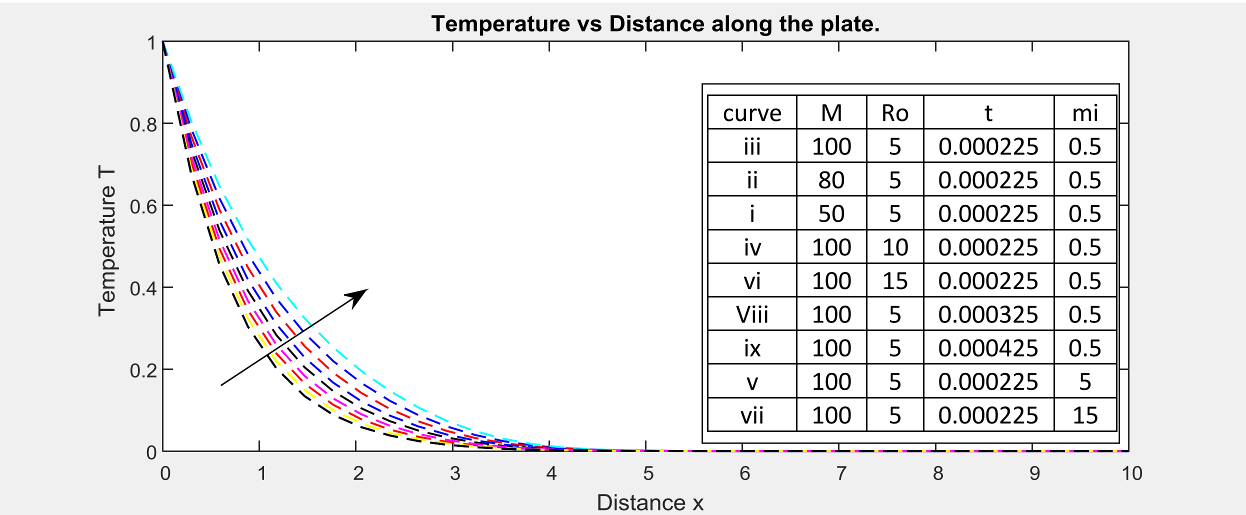

Fig 4: Temperature profiles for different values of Magnetic Parameter M, Rotation parameter Ro, time t and Hall parameter mi, for heat source and positive Grashof number.

From figure 4 it is noted that an increase in the magnetic parameter causes decrease in the temperature profiles. An increase in rotation parameter causes an increase in temperature profiles. An increase in time causes an increase in the temperature profiles. An increase in the Hall parameter mi causes an increase in the temperature profiles. Increasing mi decreases the conductivity of the fluid, resulting in an increase in Joule heating, leading to a thicker thermal boundary layer

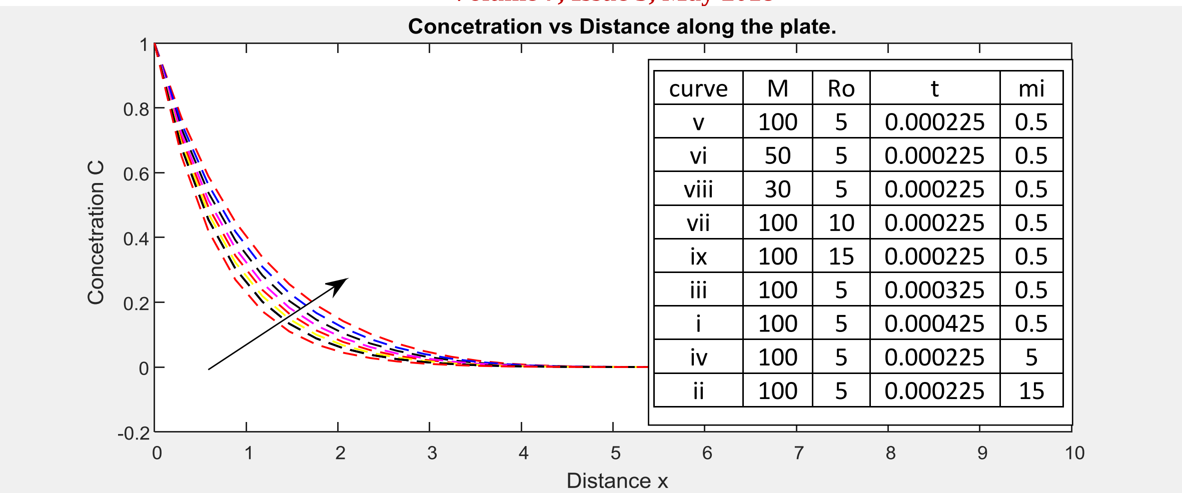

Fig 5: Concentration profiles for different values of Magnetic Parameter M, Rotation parameter Ro, time t and Hall parameter mi, for heat source and positive Grashof number.

From figures 5it is noted that an increase in the magnetic parameter causes an increase in the concentration profiles. An increase in rotation parameter causes an increase in concentration profiles. An increase in time causes a decrease in the concentration profiles. An increase in the Hall parameter mi causes decrease in the Concentration profiles.

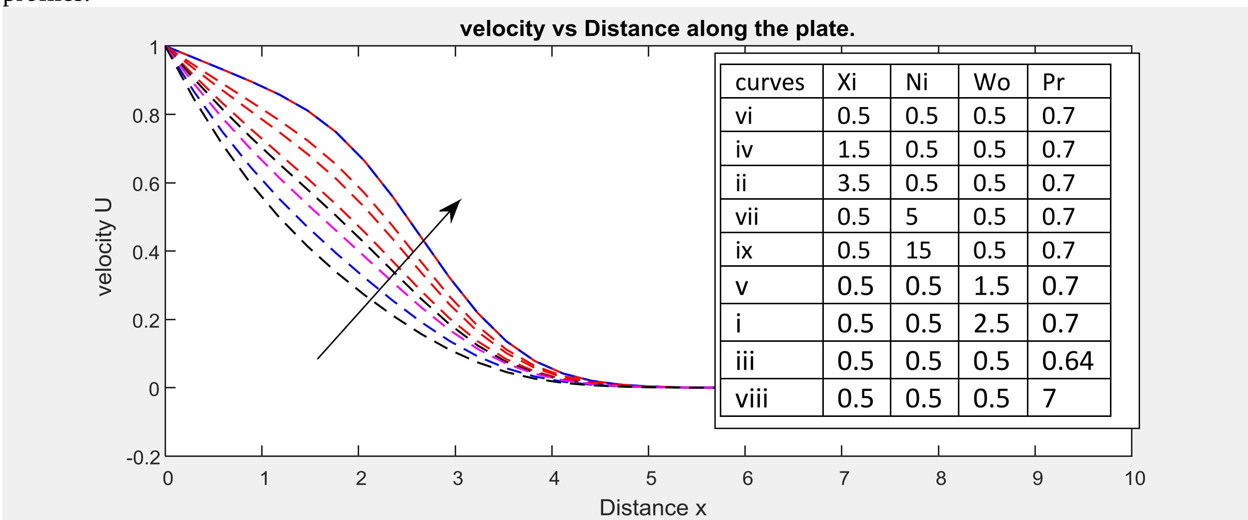

Fig 6: Primary velocity profiles for different values permeability parameter X_i , radiation parameter N, suction Parameter Wo, Prandtl number Pr for heat source and positive Grashof number.

From figure 6 and 7it is noted that an increase in the Permeability Parameter Xi leads to a decrease in the velocity. In the presence of a heat source, an increase in the Radiation Parameter N leads to an increase in the magnitude of both primary and secondary velocity. An increase in suction parameter causes a decrease in both primary and secondary velocity profiles. Increasing prandtl number leads to an increase in both primary and secondary velocity profiles.

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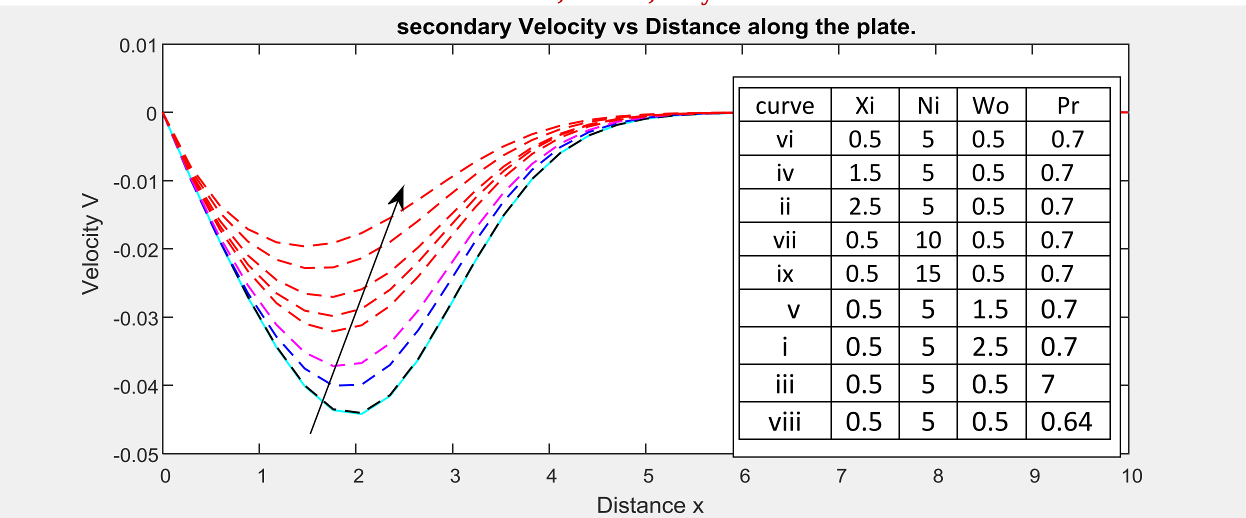

Fig 7: Secondary velocity profiles different values permeability parameter X_i , radiation parameter N, suction Parameter Wo, Prandtl number Pr for heat source and positive Grashof number.

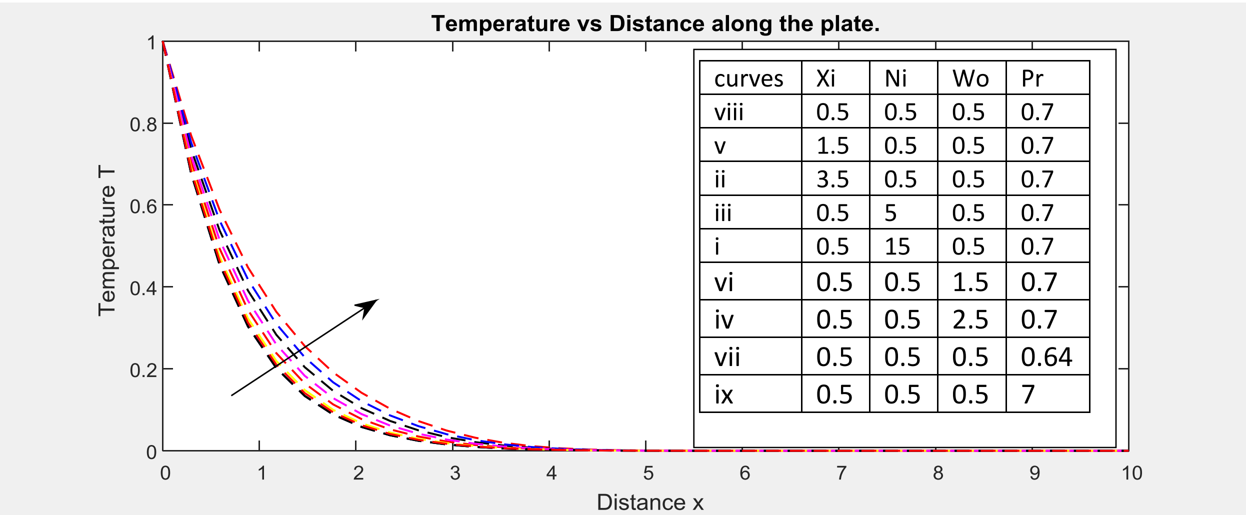

Fig 8: Temperature profiles for different values permeability parameter X_i , radiation parameter N, suction Parameter Wo, Prandtl number Pr for for heat source and positive Grashof number.

From figure 8 it is noted that an increase in permeability Xi causes a decrease in the temperature profiles. An increase in Radiation parameter causes a decrease in temperature profiles. An increase in suction parameter causes a decrease in temperature profiles. An increase in Prandtl number causes an increase temperature profiles.

Figure 9 it is noted that an increase in Xi reduces the rate of species transportation from the surface of the contracting sheet, leading to an increase in the concentration profiles. An increase in radiation parameter leads to an increase in the concentration profiles. An increase in suction parameter causes an increase in concentration profiles. An increase in Prandtl number causes a decrease in concentration profiles. Prandtl number Pr, is a measure of relative importance of viscosity and thermal conductivity of the fluid. As Pr increases, the viscous effects increase thereby reducing species molecular activity. This leads to decrease in mass average velocity that in turn leads to a decrease in concentration profiles.

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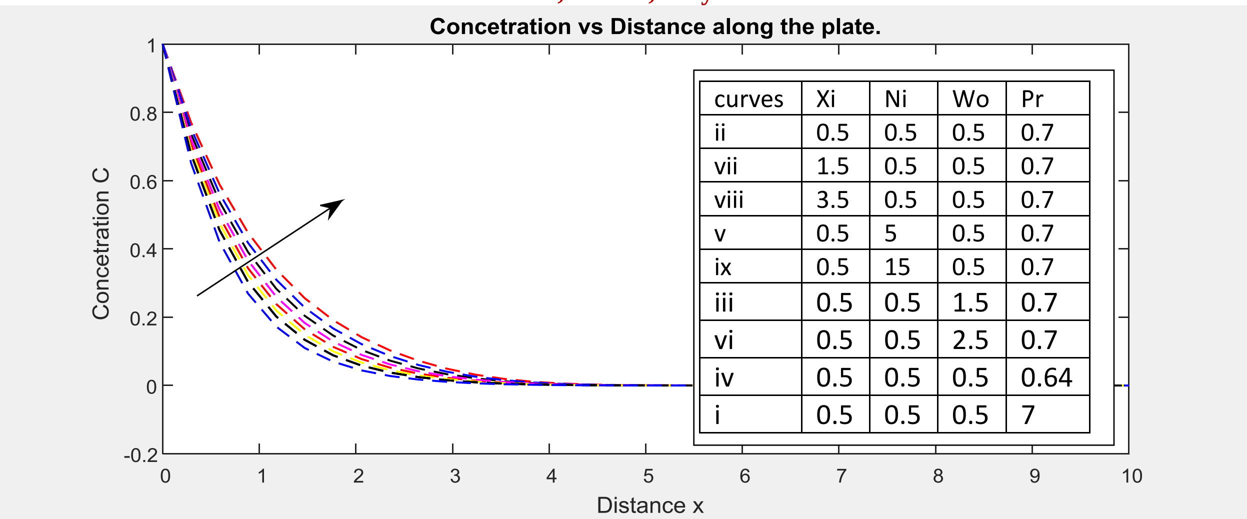

Fig 9: Concentration profiles for different values permeability parameter X_i , radiation parameter N, suction Parameter Wo, Prandtl number Pr for heat source and positive Grashof number.

V. CONCLUSION

The study of velocity field, variation of temperature and concentration past contracting rotating porous medium has been carried out. The PDE's governing the flow is highly non-linear and coupled, and the equations have been solved by using the finite difference method. Codes are run for small values of time and there is no significant difference in the results obtained. Some of the findings are:

i. Primary velocity rises with an increase in time, radiation parameter, Hall parameter and Prandtl number.

ii. Primary velocity reduces with an increase Hartman number, permeability parameter, Rotation parameter, suction parameter.

iii. Secondary velocity rises with an increase in time, radiation parameter, Rotation parameter, Hall parameter and Prandtl number.

iv. Secondary velocity reduces with an increase in Hartman number, permeability parameter, suction parameter.

v. Temperature rises with an increase in time, Rotation parameter, Hall parameter and Prandtl number

vi. Temperature reduces with increase Hartman number, permeability parameter, suction parameter and radiation parameter.

The present work can provide basis for further research by including flow that involves non-Newtonian fluids.

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NOMENCLATURE

Roman Symbol	**Quantity**
c_s	Concentration susceptibility, $[kmolm^3]$
g	Acceleration due to gravity vector, [ms^{-2}]
H	Magnetic flux density along the x- axis [wbm^{-2}]
h	Specific enthalpy, [Jkg^{-1} k^{-1}]
J	Current density, [Am^{-2}]
L	distance between vertical sheets, [m]
Q_o	Heat generation constant, $[\mathrm{W\ m^3}]$
k_T	Thermal conductivity of porous medium, [wm^{-1} k^{-1}]
T	Absolute free temperature of the fluid, [k]
T_∞	Absolute temperature of the upper sheet [k]
T_w	Absolute temperature of the contracting sheet, [k]
c	Contracting rate, [s^{-1}]
C_p	Specific heat at a constant pressure, [Jkg^{-1} k^{-1}]

k_p Darcy permeability, [m^2]

k^* Mean absorption coefficient of the fluid, $[m^{-1}]$

D_m Molecular diffusion coefficient, $[m^2 s^{-1}]$

D_T Thermal diffusion coefficient, $[kgm^{-1}s^{-1}K^{-1}]$

Q Dimensionless heat source

GREEK SYMBOL QUANTITY

β Volumetric co efficient of thermal expansion, [k^{-1}]

β^* Coefficient of thermal expansion due to concentration gradient, [k^{-1}]

σ^* Stefan-Boltzmann constant, $[Wm^{-2}K^{-4}]$

ρ Fluid density, [kgm^{-3}]

μ Coefficient of viscosity, [$kgm^{-1}s$

Ω Angular velocity, [s^{-1}]

σ Electrical conductivity, [$\Omega^{-1}m^{-1}$]

μ_e Magnetic permeability, $[\mathrm{Hm}^{-1}]$

∇ Gradient operator $i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

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AUTHOR BIOGRAPHY

Samuel kanyi Muondwe obtained his Msc. In Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya in 2014 and PhD in Applied Mathematics is in progress at

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JKUAT. Presently he is working as an assistant lecture at Dedan Kimathi University of Technology. He has published six papers in international Journals. His Research area is in MHD and Fluid Dynamics.

Professor Mathew Ngugi Kinyanjui Obtained his MSc. In Applied Mathematics from Kenyatta University, Kenya in 1989 and a PhD in Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya in 1998. Presently he is working as a professor of Mathematics at JKUAT where he is also director of Post Graduate Studies. He has published over fifty papers in international Journals. He has also guided many students in Masters and PhD courses. His Research area is in MHD and Fluid Dynamics.

Professor David Theuri. Obtained his Msc. In Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya in 1992 and PhD in Applied Mathematics from JKUAT 2003. Presently he is working as a professor of Mathematics at JKUAT. He has published over ten papers in international Journals. He has also guided many students in Masters and PhD courses. His Research area is in MHD and Fluid Dynamics.

Dr. Kang'ethe Giterere obtained his MSc. In Applied Mathematics from Jomo Kenyatta University of Agriculture and Technology (JKUAT), Kenya in 2007 and a PhD in Applied Mathematics from the same university in 2012. Presently he is working as a Lecturer at JKUAT. He has published six papers in international journals and guided many students in Masters courses. His area of research is MHD and Fluid Dynamics.