DETAILED STRUCTURE OF PIPE FLOW WITH WATER HAMMER OSCILLATIONS

P. N. Kioni

Department of Mechanical Engineering, Jomo Kenyatta University of Agriculture and Technology, Nairobi, Kenya

ABSTRACT

Herein, the evolution and detailed structure of velocity and pressure fields of an oscillating axi-symmetric pipe flow arising from a rapid closure of a valve has been determined through the solution, by the finite volume technique, of the full Navier Stokes equations. The method correctly predicts the distortion of the pressure waveform. The two-dimensional solution obtained confirms the one-dimensional solution obtained by the method of characteristics of the two-equation model incorporating frequency dependent friction typically used in modelling of transient pipe flows.

The velocity field following valve closure is seen to consist of the initial velocity profile upon which is superimposed an oscillating velocity component. The oscillating component is uniform in a pipe cross-section, at any instant. However, the profiles of initial steady state flow component and the uniform oscillating flow component are progressively distorted as the cumulative distance traversed by the wave increases.

1.0 INTRODUCTION

Transient flow, and associated wave propagation, is a frequent occurrence in pipe systems. In some cases, the feature is imposed on a flow by virtue of the characteristics of an appliance, such as a reciprocating pump, connected to the system, and operations such as pump starts and pump trips. In other cases, such as in the water-hammer phenomenon, the feature is a natural response of an entire system to temporal change at a boundary or at a critical section of a flow. Under conditions of transient flows, large pressure surges, may be generated and they may be destructive to the pipe systems. Thus, in design of pipe systems, such conditions are avoided or a system is designed to withstand the most adverse pressure conditions it may experience. The need to accurately predict the magnitude of such pressure surges and the conditions under which they occur has been a major driving force in the study of transient flows.

The governing system of equations which is normally the basis of numerical analysis of transient flow in pipe systems consists of two equations describing the conservation of mass and of linear momentum. The development of these equations is well documented by Wylie and Streeter (1983). In their most general form, they are non-linear hyperbolic partial differential equations in which the dependent variables are pressure and the mean flow velocity at a flow cross-section, whereas the independent variables are time and the distance along the axis of the flow conduit. The pair of equations has been solved for most applications using the method of characteristics which has been the most popular technique with major writers in the field, Wylie *et al.* (1983), Chaudhry (1987) and Wylie (1996).

In the application of the two-equation model to transient flows, it is necessary to modify the friction term to incorporate the dependence of fluid friction on the rate of change of velocity. Zielke (1968) has given a suitable friction term model for laminar transient pipe flow. The friction models for turbulent transient flows have been given by Vardy *et al.* (1991 and 1994).

This one-dimensional approach makes the numerical analysis of long pipe networks tractable and has been successful in providing the key information required in the design of a large variety of engineering systems in which flow transients are important. Further insight into the way in which the pressure field is coupled to the velocity field, and knowledge of how the flow field interacts with the pipe system devices, for example valves, may be obtained through two or three-dimensional analysis of the flow. However, the two or three-dimensional analysis requires a number of grid points per unit length of the pipe and, therefore, may be applied only to short pipe sections in the areas of interest.

Presented in this paper is a model for obtaining the detailed pressure and velocity field solutions for the water-hammer phenomenon for an axisymmetric pipe flow. The model consists of the full Navier-Stokes equations, the mass conservation equation and a compressibility equation. These are solved using a finite volume technique. The model is applied to obtain a solution for laminar flow field subsequent to sudden closure of a valve in a pipeline. This solution is compared to the one-dimensional solution obtained for the two-equation model incorporating frequency dependent friction, see Zielke (1968), and solved by the method of characteristics.

2.0 GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The independent variables are the time t and the spatial coordinates x and r of a cylindrical coordinate system, in which x is aligned with the pipe axis. The dependent variables are the axial and radial velocity components u and v, respectively, pressure p and density r. Thus, the conservation equations are for

- mass

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho v)=0; \qquad \text{.....(1)}$$

- x–momentum

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u^2)+x\frac{1}{r}\frac{\partial}{\partial r}(r\rho vu)=-\frac{\partial p}{\partial x}$$
$$+\frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x})+\frac{1}{r}\frac{\partial}{\partial r}(r\mu\frac{\partial u}{\partial r})+\frac{\mu}{r}\left\{\frac{\partial}{\partial x}(r\frac{\partial u}{\partial x}-\frac{2}{3}(\nabla .V))+\frac{\partial}{\partial r}(r\frac{\partial v}{\partial x})\right\}; \qquad \text{....(2)}$$

- r–momentum

$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho uv)+x\frac{1}{r}\frac{\partial}{\partial r}(r\rho v^2)=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x})+\frac{1}{r}\frac{\partial}{\partial r}(r\mu\frac{\partial v}{\partial r})+$$
$$\frac{\mu}{r}\left\{\frac{\partial}{\partial r}(r\frac{\partial v}{\partial r}-\frac{2}{3}(\nabla .V))+\frac{\partial}{\partial x}(r\frac{\partial u}{\partial r})-\left[\frac{2v}{r}-\frac{2}{3}(\nabla .V)\right]\right\}.$$

....(3)

Closure of the system of governing Eqs (1) - (3) is achieved through the compressibility equation, namely,

$$\frac{\partial p}{\partial \rho}=\frac{K}{\rho} \qquad \text{.....(4)}$$

where K is the bulk modulus of a fluid. In Eqs (2) and (3), $\nabla .V$ is defined as

$$\nabla .V=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}. \qquad \text{.....(5)}$$

2.1 Assumptions

The following assumptions have been made:

(i) The flow is laminar.

(ii) The dynamic viscosity is constant.

(iii) There are no body forces.

(iv) The effective bulk modulus, K, is constant.

(v) The pipe is rigid. Hence there is no change in volume of the computational space due to change of pressure.

(vi) There is no separation of liquid column or formation of vapor cavities.

(vii) The flow is axi-symmetric.

2.2 Boundary Conditions

Solution of the above system of governing equations requires suitable boundary conditions to be imposed. These are specified as given below for the pipe flow configuration illustrated in Figure 1. The no slip condition, $u = v = 0$, is imposed at the wall, r = R. At the line of axi-symmetry, r = 0, the condition imposed is v = 0, $\partial u/\partial r = 0$, and $\partial p/\partial r = 0$. At the inlet, $x = 0$, the pressure is specified. At the outlet, $x = L$, the velocity is specified during the valve closure, thereafter, there is no flow across the boundary and the no-slip condition is applied.

2.3 Discretisation and Numerical Solution of the System of Governing Equations

The discretised form of the system of governing equations, corresponding to equations (1) - (3), are second-order accurate in space, and are obtained following the finite volume formulation as described in Patankar (1980). The Euler implicit scheme is applied for the temporal differencing and a hybrid of upwind and central difference scheme is used for the spatial derivative terms. A collocated grid, in which the velocity u and v, density r, and pressure p are evaluated at the same cell centre, is used instead of a staggered grid. This is achieved by approximating the mass fluxes at cell interfaces following the procedure in Rhie and Chow (1983).

The compressibility equation is discretised on the Euler implicit scheme to give the following equation;

$$\left(\rho^{n+1} - \rho^{n}\right) = \left(p^{n+1} - p^{n}\right)\frac{\rho^{n}}{K} \qquad (6)$$

Equation (6) is used in the approximation of the time derivative of density in the discretized continuity equation and in updating density field, ρ^{n+1}, every time a new pressure field, p^{n+1}, is evaluated. The superscripts n and $n+1$ denote the times t and $t+1$, respectively.

The discretised system of the governing equations is solved following PISO (pressure implicit with splitting of operators) Issa (1990), which is a pressure based non-iterative procedure and which is, in many ways, similar to SIMPLE method (Patankar, 1980). The procedure is based on an operator splitting concept in which evaluation of the solution at each time step involves a predictor step followed by two corrector steps. In the predictor step the discretised momentum equations are each solved implicitly to obtain a preliminary velocity field solution. In this predictor step, the pressure field is taken to be that determined at the end of the previous time-step. The preliminary velocity field solution does not satisfy mass conservation equation. Mass conservation is enforced in the corrector steps. In the first corrector step the preliminary velocity field solution is utilised in the implicit solution of the discretised pressure equation to obtain the pressure field, and subsequently, the mass fluxes at the cell interfaces. These cell interface fluxes satisfy the mass conservation equation. Using the new pressure field and the new mass fluxes, an updated velocity field and density field are obtained. In the second corrector step, the updated solutions are used in the determination of final and more accurate solutions for the pressure field, interface mass fluxes, velocity and density fields as done in the first corrector step.

3.0 RESULTS AND DISCUSSIONS

The flow configuration consists of a pipe which is connected to a reservoir at one end and to a valve at the other end. Throughout this simulation the flow is laminar. The initial condition for the simulation is a steady state pipe flow in which the valve is fully open. The valve is closed over a short period t_c measured from the onset of the closure. The closure time t_c used is short enough to ensure that the Jowkowsky pressure rise, that is, the possible maximum pressure rise, is attained.

The principal objective in this study is to obtain two-dimensional solutions of the flow problem by finite volume technique. However, for purposes of validating the finite volume procedure, solutions are also obtained by the method of characteristics.

The two-equation model solved by the method of characteristics (MoC) incorporates the frequency dependent friction, which is implemented as given by Zielke (1968). The problem solved is that considered by Zielke (1968) and for which experimental data is given by Holmboe and Rouleau (1967). Hence, the flow is in a rigidly held pipe of diameter is 25.4 mm, length 36.088 m, kinematic viscosity $39.67\ cm^2/s$, and a wave speed, 'a', of 1324.4 m/s.

The MoC solution obtained in this work is shown in Figure 2 where it is compared with the experimental data reported in Holmboe and Rouleau (1967). Non-dimensional pressure rise $p/(a\rho U_o)$ and non-dimensional time ta/L are used. The symbol U_0 denotes the initial mean velocity at the valve. It is seen that the MoC model correctly predicts the pressure waveform. In the subsequent presentation, the one dimensional MoC solutions are used for validating the finite volume technique (FVT).

In the set-up considered above, the pipe is rather long for the problem to be conveniently modelled using the FVT, hence a direct comparison of the FVT solution to the experimental data is not possible. With the FVT, solutions are determined for pipe of length 1.4 m and diameter 10 mm and a fluid of dynamic viscosity $35.70\times10^{-3}\ kg/ms$, density $900 kg/m^3$, and bulk modulus $1.579\ GN/m^2$. Hence, the properties are those of the fluid used in Holmboe and Rouleau (1967).

In the finite volume case, the process of valve closure is simulated by reducing the velocity from the steady state value to zero following the complementary error function in which time is the variable. The complementary error function is used in order to provide both the steep change of velocity required in sudden valve closure and stability for the numerical process. Preliminary analysis indicates that, for a given closure time, the subsequent flow phenomenon is not sensitive to the closure function. At the inlet boundary, the Dirichlet condition is applied for velocity and the pressure is the initial static pressure head. At the outlet boundary, no pressure specification is required since the velocity is known.

To ascertain that the results are independent of the grid, numerical solutions were obtained on a mesh consisting of 31×101 and 51×151 points in the radial and axial coordinates, respectively, with a time step of 2 microseconds. On the latter mesh, a solution was obtained with a time step of 0.2 microsecond and, thus, it was ascertained that for a time step of 2 microseconds, the solutions are independent of the time step size. The solutions so obtained are shown in Figure 3. In this figure, pressure time-histories are plotted for (a) a point at the valve and (b) a point mid-way along the pipe. It is seen that there is virtually no difference between the solution obtained on the 151×51 mesh and that obtained on the 101×31 mesh.

An additional solution was obtained for a pipe of length 0.675 m so as to determine the solution's sensitivity to pipe length. This solution is shown in Figure 4. (The pipe length of 0.675 m and above is sufficiently long to allow full development, within the length, of the oscillating pipe flow; this was confirmed in numerical experiments in which simulations were carried out for different pipe lengths.)

It is seen in the pressure time histories shown in Figure 3 for pipe length of 1.4 m that there is good agreement between the solution obtained by the MoC and that obtained by the FVT. Specifically, the following is noted. The Jowkousky pressure rise, given by $\Delta p = a\rho U_o$, is obtained. The wave speed obtained in the finite volume solution is 1324 m/s, and this is equal to the expected wave speed determined from the input data using the formula $a = \sqrt{K/\rho}$. It is also observed that the pressure waveform is a bit distorted in the following three ways. First, during and immediately following valve closure, the pressure rises steeply to approximately the Joukousky's value and then marginally for some time before dropping steeply. Second, in subsequent oscillations, the rate of change of pressure decreases as it approaches the maximum or minimum values. However, in a half cycle, the maximum rate of change occurs immediately after the maximum or minimum value has been attained. Third, there is a progressive decrease of amplitude and an increase in waveform distortion in the subsequent cycles. For the set-up considered, the extent of distortion of the pressure waveform is dependent on the cumulative distance traversed by the wave. Thus, the distortion per cycle is greater in the configuration with longer pipes, as seen in Figure 4.

Shown in Figure 5 is the evolution of the velocity profile at a cross-section mid-way along the pipe for two cycles of the velocity-time history. Each of the two cycles cover the period between (a) points A and B, and (b) points C and D shown in Figure 3(b). Hence, they are, respectively, in the first and the third cycles of the oscillations following the valve closure. The actual velocity profiles $u(r)$ are plotted in parts (i), whereas in parts (ii) of the figure are the profiles of the velocity relative to the initial steady-state velocity, that is, $u(r)$ "$u_0(r)$. In this section of the pipe, the radial component of velocity is zero. In the plots the parabolic shape of the initial velocity is evident. It is seen that at any instant subsequent to the valve closure, the velocity in a pipe cross-section is made up of the initial steady state profile upon which is superimposed an oscillating component. The oscillating component is initially uniform in a pipe cross-section. It is always opposite in direction to the initial flow direction, and its lowest magnitude in a cycle is zero. However, the no-slip condition at the pipe wall causes distortion of the profiles. This distortion is most pronounced in a thin layer at the pipe wall. The distortion increases with time as the cumulative distance traversed by the wave increases.

4.0 CONCLUSIONS

In this paper a numerical model for two and three-dimensional solution of transient flow problems has been presented. The model consists of the Navier-Stokes, continuity and the compressibility equations as the system of governing equations which are solved using a finite volume technique. In this paper, the model has been applied to obtain detailed solution of the flow field following sudden closure of a valve in a pipeline. The model correctly predicts the pressure waveform and the distortion thereof associated with the varying influences of the inertial and frictional forces. The results agree with the one-dimensional solution obtained by the method of characteristics for the two-equation model in which a frequency dependent friction term is incorporated.

It has been found that the velocity profile at any section, away from the valve, is the sum of the initial steady state velocity profile, before the onset of the valve closure, and an oscillating velocity component. The oscillating velocity component has a uniform profile across a pipe section at any instant. The oscillating component is the result of the pressure wave propagating back and forth in the pipe. The two components of the velocity are progressively distorted as the cumulative distance traversed by the wave increases. The model presented in this paper may be used as a tool for numerical study of detailed velocity and pressure flow fields in a short pipe section encompassing a pipeline device or a region in which an event of interest occurs.

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Nomenclature

L - pipe length
R - pipe radius
K - bulk modulus of fluid
U_o - initial mean velocity at the valve
a - wave speed
g - body force
p - pressure
t - time coordinate
u - axial velocity component
v - radial velocity component
x - axial space coordinate
r - radial space coordinate

Greek and Other symbols

∇ - divergence operator
Dp - maximum pressure rise
μ - coefficient of dynamic viscosity

r - density

Subscripts

c - identifies valve closure point

o - identifies initial value

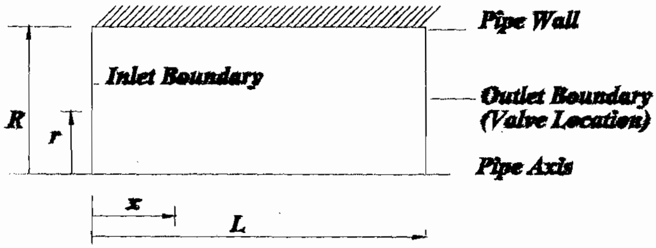

Figure 1: An illustration of the flow configuration

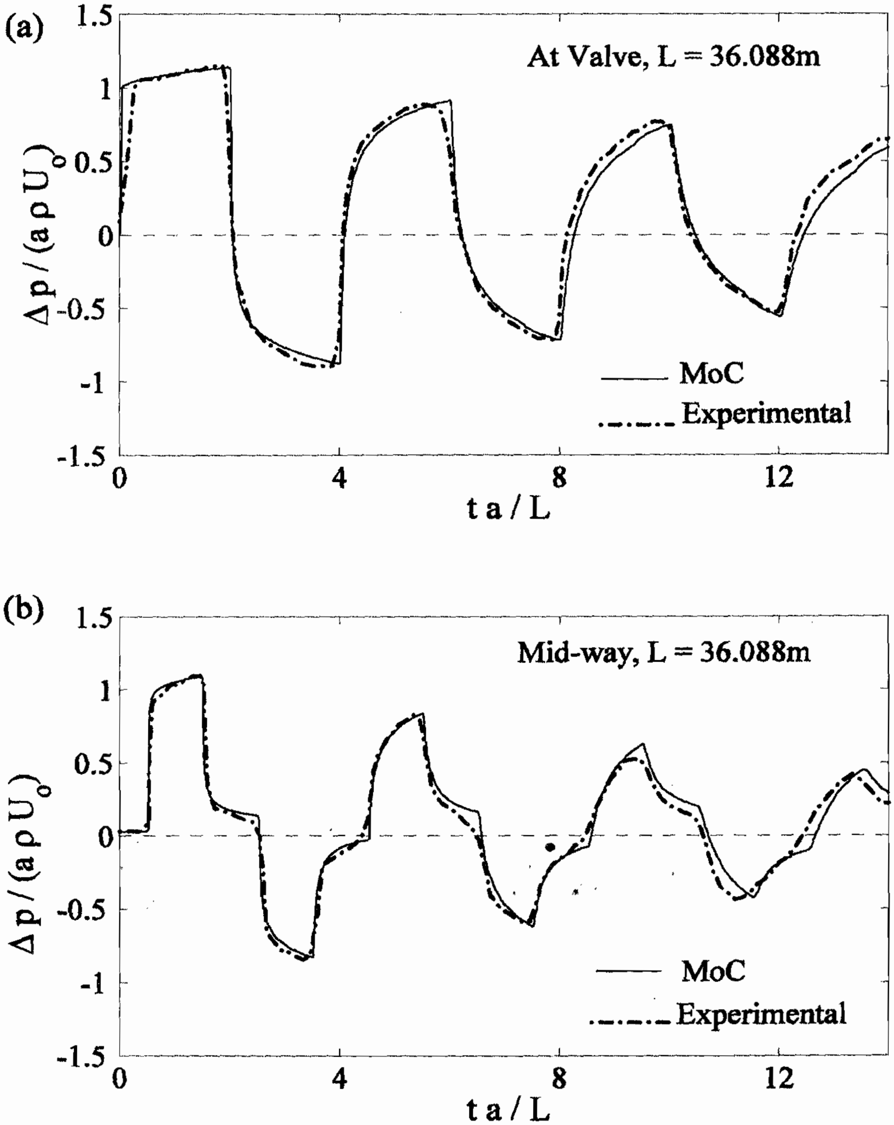

Figure 2: Experimental data and method of Characteristic (MoC) solutions for the pressure time history for a 36.088 m long pipe (a) at the valve and (b) mid-way along the pipe

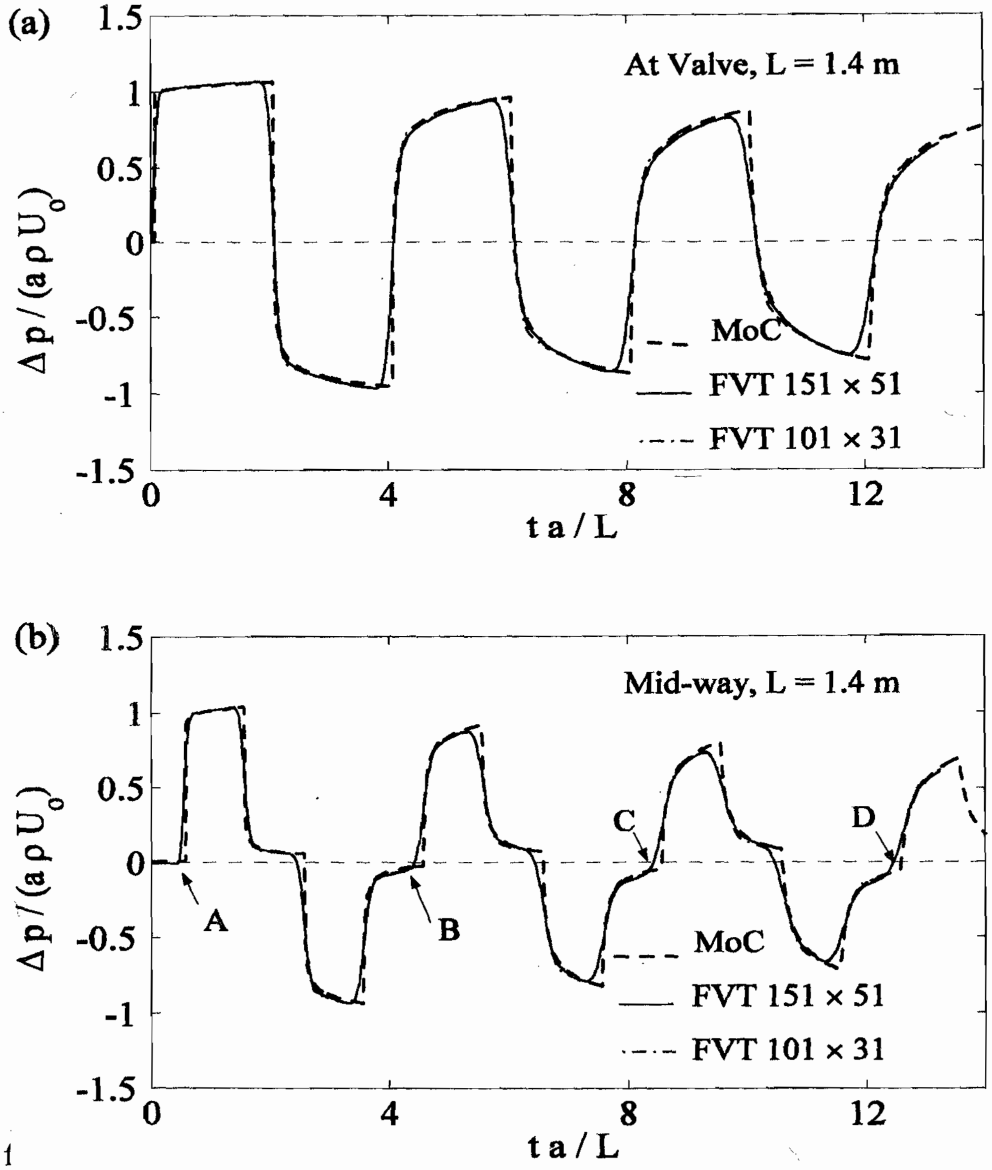

Figure 3: Pressure time history MoC solution and Finite-Volume Technique (FVT) solutions, computational mesh 151×51 and 101×31, for a 1.4 m long pipe (a) at the valve and (b) mid-way along the pipe

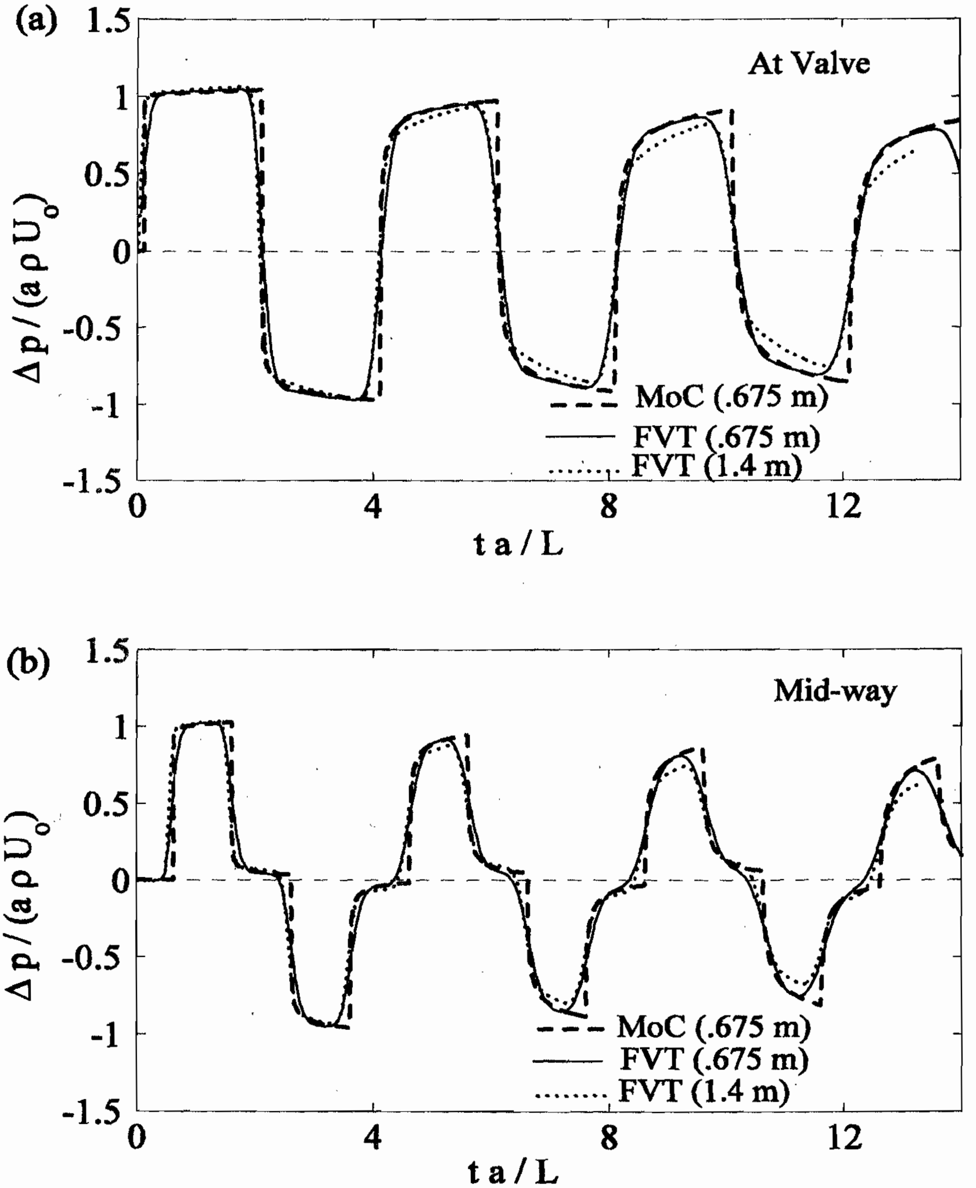

Figure 4: Pressure time history MoC solution, for a 0.675 m long pipe, and Finite-Volume Technique (FVT) solutions, for 0.675 m and 1.4 m long pipes,(a) at the valve and (b) mid-way along the pipe

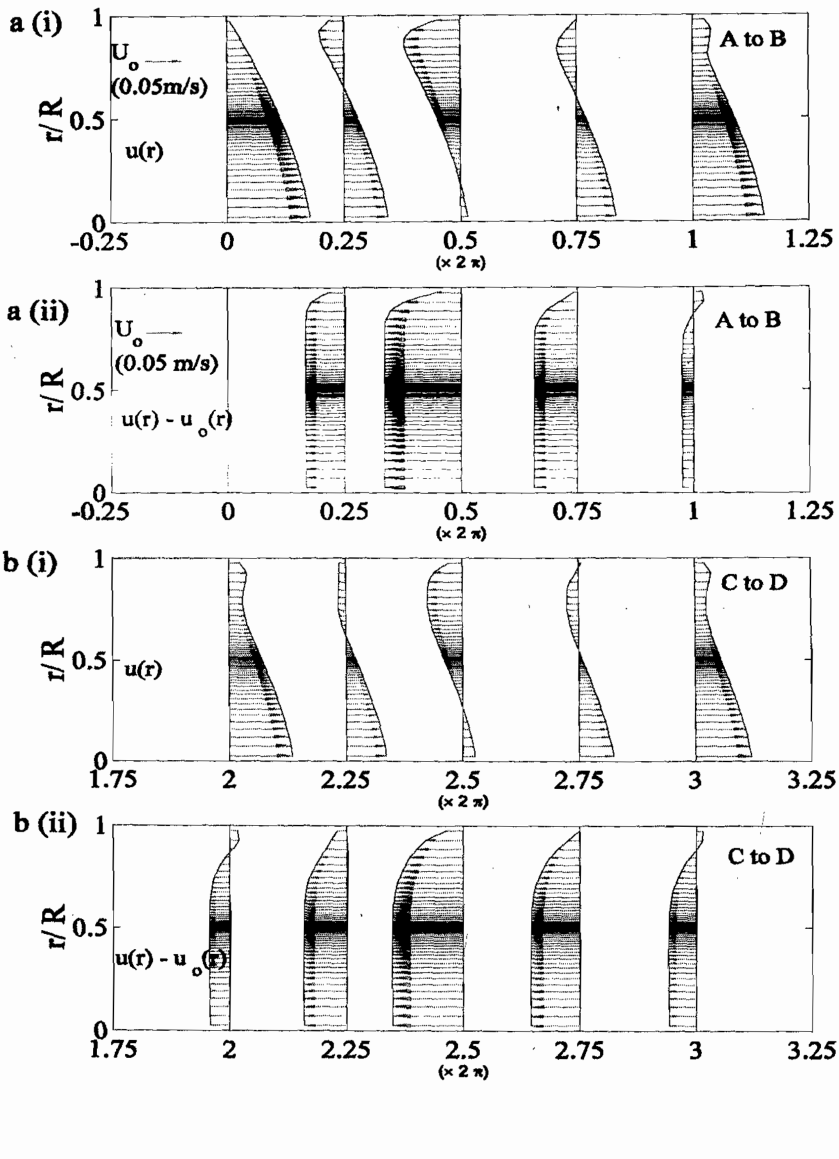

Figure 5: FVT velocity field solutions midway along the 1.4 m pipe shown as (i) overall velocity profile and (ii) velocity profile relative to initial steady state profile at different times in two cycle between (a) points A and B, and (b) points C and D shown in Figure 3