INSTRUCTIONS:

- This question paper has **FIVE** questions.
- Answer Question **ONE** and two other questions.

1. (a) Sketch the following signals over an appropriate interval (4 marks)
   i. $\sin(2\pi t)e^{-t}u(t)$ where $u(t)$ is the unit step
   ii. $u(-t+1)$, where $u(t)$ is the unit step.

   (b) Consider a differentiator whose response $y(t)$ to an input $x(t)$ is given by

$$y(t) = \frac{dx(t)}{dt}$$

   i. Verify that the system is linear (4 marks)
   ii. Verify that the system is time invariant (2 marks)
   iii. Compute and sketch the output when (4 marks)
      A. $x(t) = r(t)$, where $r(t)$ is the unit ramp.
      B. $x(t) = \frac{t^2}{2}u(t)$, where $u(t)$ is the unit step.

   (c) The impulse response of a linear time invariant system is given by $h(t) = u(t) - u(t-1)$, where $u(t)$ is the unit step.
   i. Sketch the response of the system to $x(t) = \delta(t)$ (3 mark)
   ii. If the input to the system is given by $x(t) = u(t) - u(t-1)$ where $u(t)$ is the unit step, determine and sketch $y(t)$ over an appropriate interval. (5 marks)

   (d) Consider the rectangular pulse defined as follows

$$p(t) = \begin{cases} 1 & |t| \leq \frac{T}{4} \\ 0 & \text{otherwise} \end{cases}$$

   i. Compute the Fourier transform of $p(t)$. (3 marks)
   ii. Sketch the Fourier transform of $p(t)$. (3 marks)
   iii. From your sketch, what is the null-to-null bandwidth of $p(t)$.(2 marks)

2. (a) Consider the periodic waveform shown in Figure 2a.
   i. What is the period? (2 marks)
   ii. Determine the Fourier series expansion of $x(t)$. That is determine x_n in the expression $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j\frac{2\pi n}{T_0}t}$. (4 marks)
   iii. Derive the trigonometric Fourier series representation of the signal. That is compute a_0, a_n and b_n in the expression below. (6 marks)

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2\pi nt}{T_0}\right) + b_n \sin\left(\frac{2\pi nt}{T_0}\right)\right)$$

   iv. From the expression for x_n in 2(a) and the expressions for a_0, a_n and b_n in 2(b), verify that

$$x_n = \frac{a_n - jb_n}{2} \qquad (2marks)$$

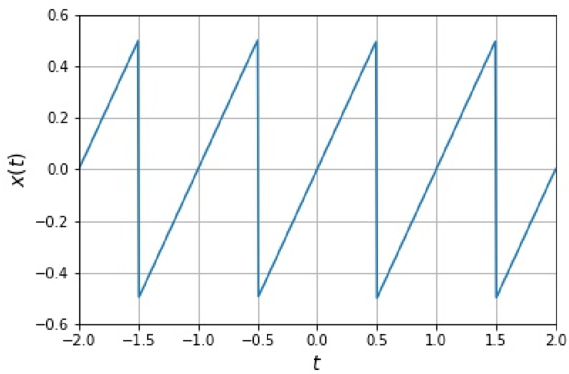

Figure 2a: Periodic waveform

(b) Consider the following signal

$$x(t) = \cos(3\pi t) + \cos(7\pi t)$$

Determine and sketch the output of the following filters when $x(t)$ is the input (6 marks)

i.

$$H_1(f) = \begin{cases} 1 & |f| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

ii.

$$H_2(f) = \begin{cases} 1 & 1 \leq |f| \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

iii.

$$H_3(f) = \begin{cases} 1 & |f| \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

3. (a) The frequency response of a filter is given by $H(f) = \mathsf{sinc}(2f)$
   i. Sketch the frequency response of the filter (2 marks)
   ii. Sketch the frequency response of $h(t)\cos(8\pi t)$ where $h(t)$ is the inverse Fourier transform of $H(f)$ (2 marks)

   (b) Explain what you understand by modulation and why it is necessary (3 marks).

   (c) Sketch the circuit of an envelope detector and explain its operation. (4 marks)

   (d) Consider the AM signal given by

$$s(t) = A_c m(t)\cos(2\pi f_c t).$$

   where $m(t)$ is a message signal given by $m(t) = A_m \cos(2\pi f_m t)$ with $f_m << f_c$
   i. Sketch this signal. (3 marks)
   ii. Give an expression for the Fourier transform of $s(t)$ (3 marks).

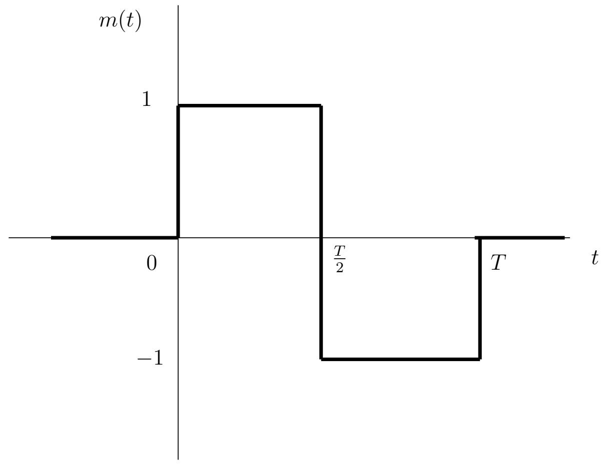

Figure 4a: Message signal m(t).

iii. Sketch the Fourier transform of $s(t)$ (3 marks).

4. (a) Consider the message signal m(t) shown in Figure 4a. m(t) is used to frequency modulate a carrier $c(t) = A_c \cos(2\pi f_c t)$ using an FM modulator of frequency sensitivity k_f.
   i. Write an expression for the instantaneous frequency of the FM signal. (2 marks)
   ii. Derive an expression for the instantaneous angle of the FM signal. (4 marks)
   iii. Sketch the resultant FM signal (4 marks)

   (b) A voice signal of bandwidth 8kHz is to be transmitted using an FM modulator with a peak frequency deviation of 24kHz.
   i. What is the deviation ratio. (2 marks)
   ii. Using Carson's rule, determine the transmission bandwidth. (3 marks)

   (c) Using an appropriate circuit diagram and relevant equations, explain the generation of FM signals using a tuned oscillator with variable capacitance. (5 marks)

5. (a) Explain the term multiplexing when applied to communication systems (4 marks)

   (b) Sketch the block diagram of a frequency division multiplexing system and explain its operation (6 marks)

   (c) Consider two independent baseband message signals $m_1(t)$ and $m_2(t)$ both of bandwidth WHz. Let $s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$ be the signal transmitted over the channel with carrier frequency f_c.
   i. Give an expression for the frequency spectrum of $s(t)$. (2 marks)
   ii. What is the bandwidth of $s(t)$. (2 marks)
   iii. Using relevant equations explain how you would recover $m_2(t)$ at the receiver. (6 marks)