



DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

University Examinations 2021/ 2022

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE IN ELECTRICAL ENGINEERING, MECHANICAL ENGINEERING, CIVIL ENGINEERING, BED CIVIL, BED EEE, BED MECHANICAL, BSC GEGIS & GIS, BSC MATHEMATICS AND MODELLING PROCESSES, BSC INDUSTRIAL CHEMISTRY, BSC POLYMER TECHNOLOGY, CHEMICAL ENGINEERING

SMA 1108: ALGEBRA

DATE: _____

TIME 2 HOURS

INSTRUCTIONS: Answer QUESTION one and any other two questions.

QUESTION ONE (30 MARKS)

- a) Simplify: $\frac{-\frac{1}{2}x^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}(1+x)^{\frac{1}{2}}}{x}$ [3 marks]
- b) Find the constant term and coefficient of the term x^6 in the expansion of $\left(3x + \frac{1}{4x}\right)^{10}$ [4 marks]
- c) If $x = \log_9 5$ and $y = \log_3 5$, find y in terms x . [3 marks]
- d) Use mathematical induction to prove that for each positive integer n , 3 is a factor of $7^n - 4^n$. [4 marks]
- e) Solve for x in the equation
- $2x^3 + 11x^2 + 17x + 6 = 0$ [4 marks]
 - $\ln x = 5$ [2 marks]
- f) Determine the set of values of x for which $2x^2 + 5x - 3 > 0$ [3marks]
- g) Rationalize $\frac{6\sqrt{5} - 4\sqrt{3}}{2\sqrt{5} - \sqrt{3}}$ [3marks]
- h) Find the values of a and b if $ax^4 + bx^3 - 8x^2 + 6$ has a remainder $2x + 1$ when divided by $x^2 - 1$ [4 marks]

QUESTION TWO (20 MARKS)

- a) The roots of the equation $3x^2 - 7x + 1 = 0$ are r and s . Find:
- $r^3 + s^3$. [4 marks]
 - the equation with integral coefficients whose roots are $r + 1$ and $s + 1$ [4marks]
- b) A ball is dropped from a height of 50 feet. It rebounds $\frac{4}{5}$ of it's height, and continues this pattern until it stops. How far does the ball travel? [5 marks]
- c) State and prove remainder theorem. [5 marks]
- d) In how many different ways can the letters in the word **FUNDAMENTALISM** be arranged in order? [2marks]

QUESTION THREE(20 MARKS)

- a) In an arithmetic progression, the fourth term is 13 and the seventh term is 22. Determine
- The first term and common difference [2marks]
 - The value of n if the n^{th} term is 100 [2marks]
 - The value of m if the sum to m terms of the series is 175. [4marks]
- b) Find the remainder when $f(x) = 3x^3 - 8x^2 - 5x + 2$ is divided by $(x - 4)$. [3 marks]
- c) Show that $2x^3 + x^2 - 13x + 6$ is divisible by $x - 2$ hence finds the other factors of the expression. [5 marks]
- d) In how many ways can a customer at a shop select 3 different types of soda from 9 available types and 6 different cakes from 10 different available cakes? [4 marks]

QUESTION FOUR (20 MARKS)

- a) Solve the equation $z^6 = -64$ for a complex number z . [6 marks]
- b) Prove by mathematical induction that $1^2 + 2^2 + 3^2 \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n . [8 marks]
- c) Use binomial theorem to expand $(3 + 4x)^{\frac{1}{2}}$ up to the term in x^4 . [6 marks]

QUESTION FIVE (20 MARKS)

- a) Show that $\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$ [7marks]
- b) How many five digit even numbers greater than 60000 can be formed from the digits 1, 2,3,4,5,6,7 and 8 if repetition are not allowed. [5 marks]
- c) Determine the line of symmetry, minimum or maximum value, x and y intercepts of $f(x) = -3x^2 + 3x + 6$. Hence sketch the curve. [8 marks]