DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2020/2021

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE
DEGREE OF BACHELORS OF ELECTRICAL ENGINEERING, MECHANICAL
ENGINEERING, CIVIL ENGINEERING, BED CIVIL,BED EEE, BED
MECHANICAL, BSC GEGIS & GIS, BSC MATHEMATICAL MODELLING,
BSC INDUSTRIAL CHEMISTRY, BSC POLYMER TECHNOLOGY
CHEMICAL ENGINEERING

SMA 1108: ALGEBRA

<u>DATE:14th DECEEMBER, 2020</u>   **DURATION: 2 HOURS   TIME: 8.30AM-10.30AM**

Instructions To Candidates
1.   Answer Question ONE and ANY OTHER TWO Questions.
2.   Show your working

<u>Question One (30 Marks)</u>

a) For a science experiment, Ajay starts with 8 milliliters of a solution at 9:00A.M. Every 20 minutes, he doubles the amount of solution. Assuming no solution evaporates, how many milliliters of solution will there be at 11:00A.M.?   (4 Marks)

b) Show that $\dfrac{\sin^2 315^0 \left(1 - \tan^2 210^0\right)}{\left(1 + \cos 120^0\right)\left(1 + \tan^2 330^0\right)} = \dfrac{1}{2}$ (3 marks)

c) Use mathematical induction to prove that for each positive integer n, 3 is a factor of
$8^n - 5^n$.   (3marks)

d) If $x = \log_9 5$ and $y = \log_3 5$, find y in terms x.   (4 Marks)

e) The first three terms in an arithmetic sequence are $\dfrac{x}{2} + 5$, $\dfrac{x}{2} + 12$, and $\dfrac{x}{2} + 19$. What is the 15th

term in the sequence?   (3 Marks)

f) Determine the values of p for which the equation $4x^2 - 2px + 2p - 3 = 0$ has real equal roots.

(4 Marks)

g) Prove by Mathematical Induction that for any natural number n, $\displaystyle\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$.   (4 Marks)

h) A rubber ball is dropped from a height of 81 m. Each time it strikes the ground, it rebounds two-thirds of the distance through which it last fell. Through what distance had it traveled from the time it was dropped until it struck the ground for the sixth time?   (2 Marks)

i) Express $z = -7 + 6i$ in polar coordinate form.   (3 Marks)

QUESTION TWO   (20 MARKS)

a) A sequence of numbers is formed by adding the corresponding terms of an A.P and G.P. The common ratio of G.P is 4. The first three terms of sequence are 8, 20 and 59. Find first term of an A.P and of the G.P   (4marks)

b) The roots of the equation $3x^2 - x - 4 = 0$ are p and q. Find without first solving this equation

   i) The value of $p^3 + q^3$.   (3 marks)

   ii) The equation whose roots are $p - \dfrac{p}{q}$ and $q - \dfrac{q}{p}$.   (4 marks)

c) Solve the equation $2^{x-2} - 2^{2-x} = 1$ giving your answer to 4 decimal places.   (4 marks)

d) When the expression $x^5 + 4x^2 + ax + b$ is divided by $x^2 - 1$, the remainder is $2x + 3$. Find the value of a and b (5 marks)

Question Three (20 Marks)

a) What is the 20th term of the arithmetic sequence $3x$, $3x + 2$, $3x + 4$, …?   (3 Marks)

b) The formula $P = 0.61x^2 - 0.041x + 3$ models the approximate population P, in thousands, for a species of fish in a local pond, x years after 1997. During what year will the population reach 24,714 fish?   (7 Marks)

c) Solve for x if:

   i. $\log_3 x + \log_9 x^2 = 6$ (4marks)

   ii. $e^{x+3} = 1.62$.   (3 marks)

d) Simplify: (3 marks)

$$\frac{-\dfrac{1}{2} x(1-x)^{-\frac{1}{2}} + (1-x)^{\frac{1}{2}}}{x^2}$$

Question Four (20 Marks)

a) Use the vertex and intercepts to sketch the graph of the quadratic function.

   $f(x) = 4 - (x-2)^2$.   (7 Marks)

b) Solve the radical equation, and check all proposed solutions.

   $\sqrt{6x + 55} = x$.   (3 Marks)

c) Find the constant term in the expansion $\left(7x - \dfrac{1}{4x^3} \right)^{19}$ (3marks)

d) How many five digit even numbers greater than 60000 can be formed from the digits 0, 1, 2,3,4,5,6,7 and 8 if repetition are not allowed.   (5marks)

e) In how many different ways can the letters in the word **INTERCOMMUNICATION** be arranged in order?   (2marks)

Question Five (20 Marks)

a) Perform the indicated operations and write the answer in the form a + b*i*, where a and b are real numbers:

i. $(3+9i)(6-4i)$. (3 Marks)

ii. $\dfrac{2-6i}{2-i}$ (3 Marks)

b) Show that $\tan 5\theta = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$. (7 Marks)

c) Prove by Mathematical Induction that for any natural number n, $\displaystyle\sum_{i=1}^{n} i = \dfrac{1}{2}\left(n+\dfrac{1}{2}\right)^2$.

(7 Marks)