DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2020/2021

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE

DEGREE OF BACHELORS OF MECHANICAL ENGINEERING, CIVIL ENGINEERING, BED CIVIL, BED MECHANICAL, BSC GEGIS & GIS, CHEMICAL ENGINEERING

SMA 2232: ORDINARY DIFFERENTIAL EQUATIONS

Date: 16ᵗʰ December, 2020   **TIME: 8.30AM-10.30AM**

QUESTION ONE (30 MARKS)

a) Differentiate between general and particular solution of a differential equation

(2 marks)

Hence find the particular solution of $y'' - 4y - 5y = 0$ $y(0) = 3$ $y'(0) = 0$

(3 marks)

b) Solve the initial value problem $\dfrac{dy}{dx} + y = \cos x$ $y\left(\dfrac{f}{2}\right) = 1$ (5 marks)

c) Find the differential equation associated with the primitive $f(x) = a_1 e^{2x} + a_2 e^{-x} + a_3$

where $a_1, a_2 \; and \; a_3$ are arbitrary constants is a solution to the equation   (4 marks)

d) Solve $y' = e^{-4x} \cos^2 5y$.   (3 marks)

e) Use inverse differential operator method to solve the differential equation.

$\dfrac{d^2 y}{dx^2} - \dfrac{dy}{dx} - 2y = 2\sin 3x$.   (5 marks)

f) Solve the homogenous equation $\left(3y^2 - x^2\right)dx - \left(2xy\right)dy = 0$ (5 marks)

g) Show that $F = xy$ is an integrating factor for the equation

$(2y^2 - 9xy)dx + \left(3xy - 6x^2\right)dy = 0$ (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the initial value problem: $\dfrac{dy}{dx} + \dfrac{3}{x}y = 2x^2$ given that $y(1) = -3$ (5 marks)

b) Use Laplace transform method to solve the initial value problem

    $y'' - y = e^{-3t}$ $y(0) = 0$ $y'(0) = 1$ (7 marks)

c) Find the power series solution in powers of x of the differential equation

    $y'' + xy' + 3y = 0$ About an ordinary point $x = 0$ (8 marks)

QUESTION THREE (20 MARKS)

a) Show that $y_1(x) = \cos 2x$ and $y_2(x) = \sin 2x$ form the fundamental set of solutions for the differential equation $y'' + 4y = 0$ (3 marks)

b) Solve the Riccatti equation $y' = \dfrac{1}{x}y^2 + \dfrac{2}{x}y + \dfrac{1}{x}$ $y(1) = -3$ (8 marks)

c) Show that $y = z \sin x$ where z is a function of x reduces the differential equation

    $\sin^2 x \dfrac{d^2 y}{dx^2} - 2\cos x \sin x \dfrac{dy}{dx} + 2y = x \sin^3 x$ to a differential equation $\dfrac{d^2 z}{dx^2} + z = x$. Solve

    the later equation and hence find y given that $y(0) = 0$ and $y'(0) = 0$ (9 marks)

QUESTION FOUR (20 MARKS)

a) Use Taylor series expansion to obtain the power series solution of

    $y'' + x^2 y' - 2 = e^x$ $y(0) = 2$ $y'(0) = -1$ (4 marks)

b) Show that the O.D.E. $6xy\,dx + (4y + 9x^2)dy = 0$ is a non-exact differential equation. Find its integrating factor hence solve it.   (6 marks)

c) Solve the equation $y'' - y' - 6y = e^{-2x}$ using the method of variation of parameters.

          (10 marks)

QUESTION FIVE (20 MARKS)

a) Solve the Bernoulli equation $y' - y = e^{2x} y^{-4}$. (5 marks)

b) Obtain the general solution of $(x-5)^2 y'' - 2(x-5) y' + 2y = \ln(x-5)^2$

(7 marks)

c) Use Laplace transform to solve the system of equations

$$\frac{dx}{dt} + \frac{dy}{dt} + x - y = 0$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} + x = 1 \qquad x(0) = y(0) = 0$$

(8 marks)