KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

University Examinations 2011/2012

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN **ACTUARIAL SCIENCE**

SMA 2493 Survival Analysis

DATE: 27TH FEBRUARY 2012 TIME: 2.00 PM – 4.00 PM

Instructions: Answer QUESTION ONE and any other TWO QUESTIONS.

QUESTION ONE (30 marks) (COMPULSORY)

(a). (i). Define the hazard rate, $h(t)$, of a random variable T denoting lifetime.
. **[1 marks]**

(ii). State the fundamental relationship that link $F(t)$, $S(t)$, $f(t)$ and μ_t. **[2 marks]**

(iii). Suppose that a group of people are subject to a hazard of death which changes linearly with age; i.e. $h(t) = \alpha + \beta t$. Derive the expression for the survivor function $S(x)$ and the pdf $f(x)$ in terms of α and β. **[5 marks]**

(b). (i). Write down the equation of the Cox proportional hazards model in which the hazard function depends on duration t and a vector of covariates z. You should define all the other terms that you use. **[2 marks]**

(ii). Explain why the Cox model is sometimes described as "semi-parametric".
. **[1 marks]**

(c). Define right censoring, left censoring, interval censoring and random censoring with the help of suitable examples. **[8 marks]**

(d). In a clinical trial, 50 patients are observed for two years following treatment with a new drug. The following data show the period in complete months from the initial treatment to the end of observation for those patients who died or withdrew from the trial before the end of the two year period.

Deaths	6,	6,	12,	15,	20,	20,	23
Withdrawals	1,	3,	5,	8,	10,	18	

(i). Calculate the Nelson-Aalen estimate of the integrated hazard function, Λ_t.
. **[6 marks]**

(ii). Hence or otherwise, estimate the probability of a patient surviving for at least 18 months after the initial treatment. **[2 marks]**

(e). List three advantages of the two-state model over the Binomial model for the estimation of transition intensities in a case where exact dates of entry into and exit from observations are known. **[3 marks]**

QUESTION TWO (20 marks)

(a). An Institute conducts tuition classes starting from 9th standard as part of their preparation for the professional entrance exams. The management of the institute is concerned with the withdrawal rates of the children and hence it is testing a new tuition method to improve the persistency rates. Data have been collected and a Cox proportional hazards model has been fitted for the hazard of students leaving the course. Symmetric 95% confidence intervals (based upon standard errors) for the regression parameters are shown below.

Covariate Confidence Interval

Course

–Engineering 0
–Medical [0.08, 0.25]

Tuition Method

–Traditional 0
–New [-0.05, 0.05]

Sex

–Boys [0.02, 0.12]
–Girls 0

(i). Write down a general expression for the Cox proportional hazards model, defining all terms that you use. **[2 marks]**

(ii). State the regression parameters for the fitted model. **[2 marks]**

(iii). Describe the class of children to which the baseline hazard applies. **[1 marks]**

(iv). Discuss the suggestion that the new tuition method has improved the chances of children continuing with the tuition classes. **[4 marks]**

(b). During a 2 year trial for a new medical treatment, 50 patients were observed after receiving the new treatment on July 2000. For those patients who died or who left the trial before 30 June 2002, a record was kept of their time spent under observation. The details are shown below:

Period under observation (in months) for patients who:

Died	Left trial
4	2
6	5
6	7
8	9
11	10
16	13
22	15
	18

(i). Calculate the Kaplan-Meier estimate of the survival function, $S(t)$. **[8 marks]**

(ii). Sketch the hazard, $h(t)$, implied by the Kaplan- Meier estimate of $S(t)$.
. **[3 marks]**

QUESTION THREE (20 marks)

(a). A large life insurance company has been selling term assurance plan for number of years and has carried out a mortality investigation for the first time recently. Mr.BK, the companys Appointed Actuary is generally satisfied with the results and has recommended that the mortality rates obtained be used for setting mortality assumptions in the future. However, Mr.BK has also suggested that the crude rates from the investigation are smoothed before using them in actuarial calculations. He has sought your opinion on the appropriate method for graduation. Briefly describe three methods of graduation that can be used, stating clearly the advantages and disadvantages of each as relevant to the insurance company. **[9 marks]**

(b). The Insurance Regulatory Authority of Kenya is concerned that a systemic increase in lapse rates may adversely affect profitability of the life insurance industry as a whole. It has been suggested that younger policyholders are more likely to lapse their policies than their older counterparts, therefore the Regulatory Authority has commissioned a study into policy lapse rates by age.

(i). Explain what is meant by Central Exposed to Risk and specify what data is needed if this is to be calculated exactly. **[3 marks]**

The Council collects the following data from a survey of all life insurance companies:

* Number of policies in-force for lives aged x next birthday on 31 December 2009, 31 December 2010 and 31 March 2011.
* Number of lapses during the period 1 January 2010 to 31 March 2011 given by age last birthday.

(ii). Derive a formula for the central exposed to risk that corresponds to the lapse data stating any assumptions that you make. **[5 marks]**

(iii). Comment on the reasonableness or otherwise of the assumptions you made in your answer to part (ii). **[3 marks]**

QUESTION FOUR (20 marks)

(a). The pdf of an exponential distribution is

$$f(x) = \lambda e^{-\lambda x}; \quad x > 0$$

Show that the hazard function is a constant λ. **[4 marks]**

(b). The likelihood function for censored data is given by

$$L = \prod_{j=1}^{k} \lambda_j^{d_j} (1 - \lambda_j)^{n_j - d_j}$$

Using $\ln L$, obtain the ML estimator of λ_j. **[4 marks]**

(c). A life insurance company carried out a mortality investigation. It followed a sample of n policyholders, observing them from their 40th birthday until either they died, or they withdrew from the investigation while still alive, or they celebrated their 45th birthday (whichever of these events occurred first).

Assuming that the force of mortality does not vary with age between exact ages 40 and 45 years, find the maximum likelihood estimate of this constant force of mortality, given the data on 20 individual in the following table:

Person Number	last age of which person was observed(years)	rate of the person
1	41.0	Died
2	42.0	Died
3	45.0	Survived
4	45.0	Survived
5	40.5	Withdrew
6	41.0	Withdrew
7	44.0	Withdrew
8	45.0	Survived
9	45.0	Survived
10	44.5	Died
11	44.0	Died
12	43.0	Died
13	42.0	Withdrew
14	45.0	Survived
15	42.5	Died
16	40.5	Died
17	41.0	Withdrew
18	42.0	Withdrew
19	43.0	Withdrew
20	43.0	Withdrew

[12 marks]

QUESTION FIVE (20 marks)

The following table refers to the survival time (in years) of the patients in a certain clinical trial. An asterisk indicates that the observation was censored. It is assumed that censoring was random, and that patients were randomly assigned to the treatment or control arms.

Control arm	3	6^*	7	8^*	8^*	9	11	13
Treatment arm	4^*	5	8^*	8^*	8^*	10	12^*	14^*

Assuming a proportional hazards model, find the estimate of the log-hazard ratio β using the Cox proportional hazards method. **[20 marks]**