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KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

University Examinations 2011/2012

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACELOR OF SCIENCE IN **ACTUARIAL SCIENCE**

SMA 2492: CREDIBILITY THEORY AND LOSS DISTRIBUTIONS

DATE: **2ND** MARCH 2012 TIME: 11.00 AM – 1.00 PM

Instructions: Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) An insurance company uses an exponential distribution to model the cost of repairing insured vehicles' that are involved in accidents .find the maximum likelihood estimate of the mean cost, given that the average cost of repairing a sample of 1,000 vehicle was 2,200. (6 marks)

b) Based on an analysis of past claims. An insurance company believes that individual claims in a particular category for the coming year will have a mean size of ksh. 5, 000, and a standard deviation of ksh.7, 500. Estimate the proportion of claims that will exceed ksh.25, 000, assuming that individual claim size conform to lognormal distribution. (7 marks)

c)

(i). Define the following terms:

a) Deductibles (1 marks)

b) Policy limits (1 marks)

(ii). Show that if $X \sim N(«, ±^2)$, then the expected amount paid per loss event can be written as:

$$\mathrm{E}\left[(X-d)_+\right] = ±¤\left(\frac{d-«}{±}\right) - (d-«)\left[1-\Phi\left(\frac{d-«}{±}\right)\right]$$

Here $¤(.)$ denotes the density of a standard normal and $\Phi(.)$ denotes the cumulative distribution function of a standard normal. (5 marks)

d) You are given that the number of claims follows a poison distribution, the claim sizes follows a gamma distribution with parameters $\sim (unknown)$ and $¯ = 10{,}000$. The number of claims and claim sizes are independent and full credibility standard has been selected so that actual aggregate losses will be with 10% of aggregate losses 95% of the times.

Using limited Fluacho-(classical) credibility determine the expected number of claims required for full credibility. (4 marks)

e) For a portfolio of policies, you are given;

i) The annual claims amount on a policy has probability density function

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$$f\left(x/\!\!-\right)=\frac{2x}{¯^{2}} \qquad 0<x<¯$$

ii) The prior distribution of ¯ has density function.

$$®\left(¯\right)=4¯^{3} \qquad 0<¯<1$$

iii) A randomly selected policy had claim amount 0.1 in year 1

Determine the Buhlmann credibility estimate of the claim amount for the selected policy in year two. (6 marks)

QUESTION TWO (20 MARKS)

a) Consider a policy limit u. Show that the expected payment can be expressed as:

$$\mathrm{E}(Y)=u-\int_0^u F_X(x)dx=\int_0^u\left[1-F_X(x)\right]dx$$

(4 marks)

b) For an insurance portfolio

i) The number of claims has the probability distribution

n	P_n
0	0.1
1	0.4
2	0.3
3	0.2

ii) Each claim amount has a poison distribution with mean 3, and

iii) The number of claims and claims amount are mutually independent.

Calculate the variance of the aggregate claims. (5 marks)

c) For a given risk, the number of claims for a single exposure period is given by a binomial distribution with $n=2, p=0.3$. The size of a claim will be 50 with probability of 50%, or 100 with probability of 20%. Frequency and severity are independent.

Determine the process variance of the pure premium for this risk. (5 marks)

d) You are given:

i) Claim sizes follows an exponential distribution with mean ¯.

ii) For 0.80 of the policies, $¯=8$

iii) For 0.20 of the policies, $¯=2$

A randomly selected policy had one claim in year 1 of size 5.

Calculate the Bayesian expected claim size for this policy in year two. (6 marks)

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QUESTION THREE (20 MARKS)

a) A health maintenance organization (HMO) currently pays full cost of any emergency room to its clients. You are given that the cost of an emergency room care has an exponential distribution with mean 1,000.
The company is evaluating the possible saving of imposing a deductible of ksh.200.per emergency room visit, to be paid by the client.

   i) Calculate the resulting loss elimination ratio due to a deductible of ksh.200 interpret this ratio.

   ii) Suppose the HMO decide to impose a per loss deductible of ksh.200 per emergency room visit, along with a policy limit of ksh.5,000 and a coinsurance factor of 80%.for every visit to the emergency room. Calculate the expected claim amount per loss event and the expected claim amount per payment event made by the HMO. (8 marks)

b) You are given ;

   i) Losses on a company's insurance policies follow a Pareto distribution with probability density function.

$$f(X|\theta)=\frac{\theta}{(X+\theta)^2} \qquad 0 < X < \infty$$

   ii) For half of the company's policies, $\theta = 1$, while for the other half $\theta = 3$.
For a randomly selected policy, losses in year one were 5.

Determine the posterior probability that losses for this policy in year 2 will exceed 8.

(7 marks)

c) You are given;

   i) The number of claims incurred in a month by any insured has a Poisson distribution with mean λ.

   ii) The claim frequencies of difference insured are independent

   iii) The prior distribution is gamma with probability density function

$$f(\lambda)=\frac{(100\lambda)^6 e^{-100\lambda}}{120\lambda}$$

   iv)

Month	Number of insured's	Number of claims
1	100	6
2	150	8
3	200	11
4	300	?

Determine the Buhlmann-Straub credibility estimate of the number of claims in month 4.

(5 marks)

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QUESTION FOUR (20 MARKS)

a) A company has determined that the limited fluctuation full credibility standard is 2000 claims if:

The total number of claim is to be within 3% of the value with probability p. using limited fluctuation credibility determine the expected number of claims necessary to obtain full credibility under the new standard. (4 marks)

b) A compound distribution S is such that $P(N=0)=0.6$, $P(N=1)=0.3$, $P(N=2)=0.1$ claim amounts are either 1unit or 2 units each with probability 0.5.derive and sketch the distribution of S . (6 marks)

c) Two types of insurances claims are made to an insurance company .For each type, the number of claims follows a Poisson distribution and the amount of each claim is uniformly distributed as follows:

Type of claim	Poisson parameter a for No. of claims	Range of each claim amount
I	12	$[0,1]$
II	4	$[0,5]$

The numbers of claims of the two types are independent and the claim amounts and the claim numbers are independent. Calculate the normal approximation to the probability that the total of claim amount exceeds 18. (10 marks)

QUESTION FIVE (20 MARKS)

a) For a portfolio of motorcycle insurance policyholders, you are given:
   i) The number of claims for each policyholder has a conditional Poisson distribution
   ii) For year 1,the following data are observed:

Number of Claims	Number of policyholders
0	2000
1	600
2	300
3	80
4	20
Total	**3000**

Determine the credibility factor Z, for year 2. (10 marks)

b) You are given :
   i) The annual number of claims on a given policy has the geometric distribution with parameter .
   ii) One third of the policies have $=2$, and the remaining two thirds have $=5$.
   A randomly selected policy had two claims in year 1.

Calculate the Bayesian expected number of claims for the selected policy in year 2. (10 marks)