



**KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY**

**University Examinations 2011/2012**

**FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**SMA 2491: ACTUARIAL MATHEMATICS III**

**DATE: 11<sup>TH</sup> AUGUST 2011**

**TIME: 2 HOURS**

**Instructions:** Answer QUESTION ONE and any other TWO QUESTIONS.

**QUESTION ONE (30 marks) (COMPULSORY)**

- (a). (i). State the conditions which are necessary for the retrospective policy value of a contract to be equal to the prospective policy value throughout the term of the policy. **[2 marks]**
- (ii). Calculate the gross reserve that should be held at the end of 10 years for a 25-year regular premium endowment assurance policy with sum assured of Ksh.175,000 payable on maturity or at the end of the year of earlier death. The policy was taken out by a 45-year old.

Expenses: 75% of the first premium and 2.5% of each subsequent premium, plus an initial expense of Ksh.1250. Assume A1967-70 select mortality and 4% interest. **[8 marks]**

- (b). (i). Briefly explain why a life insurance company will need to set up reserves for the endowment assurance contracts it has sold. **[2 marks]**
- (ii). A whole life assurance with sum assured of £100,000 payable at the end of the year of death was purchased by a life aged 30. The policy has annual premiums payable throughout life.

The basis for calculating reserves for this policy is as follows:  
Net premium method: A1967-70 ultimate, 5% interest.

Estimate the policy value at duration  $28\frac{1}{4}$  by interpolation. **[6 marks]**

- (c). Assuming the mortality of A1967-70(ultimate) for both lives, calculate the following
- (i).  ${}_3p_{45:41}$
- (ii).  $q_{66:65}$

(iii).  $\mu_{38:30}$  [6 marks]

(d). Define the following functions in words, and give an expression for each of them in terms of integral.

(i).  ${}_{\infty}q_{xy}^1$  [3 marks]

(ii).  $\bar{A}_{xy}^2$  [3 marks]

### QUESTION TWO (20 marks) (Optional)

(a). (i). Show that

$$({}_tV_x + P_x)(1 + i) = p_{x+t} \cdot {}_{t+1}V_x + q_{x+t}$$

[4 marks]

(ii). On 1 January 1985 a life office issued a number of 30 year pure endowment assurance contracts to lives then aged 35, with premiums payable annually in advance throughout the term or until earlier death. In each case, the only benefit was a sum assured of £20,000, payable on survival to the end of the term.

During 1996, 4 policyholders died out of the 580 policyholders whose policies were in force at the start of the year.

Assuming that the office uses net premium policy reserves, calculate the profit or loss from mortality for 1996 in respect of this group of policies.

Basis: mortality: A196770 ultimate  
interest: 4% per annum

[8 marks]

(b). A member of a pension scheme is aged 55 exact, and joined the scheme at age 35 exact. She earned a salary of KShs. 40,000.00 in the 12 months preceding the scheme valuation date. The scheme provides a pension on retirement for any reason of 1/80th of final pensionable salary for each year of service, with fractions counting proportionately. Final pensionable salary is defined as the average salary over the three years prior to retirement. Using the functions and symbols defined in, and assumptions underlying, the Example Pension Scheme Table in the Actuarial Tables:

(i). Calculate the expected present value now of this member's total pension.

[5 marks]

(i). Calculate the contribution rate required, as a percentage of salary, to fund the future service element of the pension.

[3 marks]

### QUESTION THREE (20 marks) (Optional)

(a). A life office issues a 3-year without profits endowment assurance policy to a life aged 62. The sum assured of KShs. 1,500.00 is payable on maturity or at end of the year of death, if within 3 years, and there are level annual premiums of KShs. 472.50 payable in advance.

The office uses the following "experience" basis:

mortality: A1967-70 ultimate  
interest: 6% per annum  
initial expenses: KShs. 20.00  
renewal expenses: KShs. 5.00 at the beginning of the 2nd and 3rd policy years.

The office's reserve basis is as follows:  
Net premium method, using A1967-70 ultimate mortality and 3% p.a. interest.

Determine the profit signature of this policy. [14 marks]

- (b). Prove that, for a whole of life assurance, the reserve at integral duration for an  $m$ -thly true premium policy (i.e.,  $P_x^{(m)}$ ) is given by

$${}_tV_x^{(m)} = {}_tV_x \left( 1 + \frac{m-1}{m} P_x^{(m)} \right)$$

[6 marks]

#### QUESTION FOUR (20 marks) (Optional)

- (a). Describe the calculation of a surrender value for a without-profit endowment assurance policy, under which level annual premiums are payable monthly in advance and cease on earlier death or surrender and the sum assured is payable immediately on death.  
Give the formula, defining carefully all the symbols that you use. [5 marks]

- (b). 12 years ago a man then aged 48 effected a without profit whole life assurance for £10,000 (payable at the end of the year of death) by annual premiums. The premium now due is unpaid. He now wishes to alter the policy so that the same sum assured will be payable at the end of the year of the first death of himself and his wife, who is 4 years older than himself. Calculate the revised office annual premium, ceasing on the first death, if the office uses the following basis for premiums and reserves.

mortality: A1967-70 ultimate, rated down 4 years for female lives,  
interest: 4% per annum,  
expenses: 3% of all office premiums including the first, with additional initial expenses of  $1\frac{1}{2}\%$  of the sum assured. (This additional initial expense is not charged again on the conversion of an existing policy, providing that the sum assured does not increase.) [15 marks]

#### QUESTION FIVE (20 marks) (Optional)

- (a). Define  $\ddot{a}_{60:50:\overline{20}|}^{(12)}$  fully in words and calculate its value using PMA92C20 and PFA92C20 tables for the two lives respectively at 4% interest. [6 marks]
- (b). A joint life annuity of 1 per annum is payable continuously to lives currently aged  $x$  and  $y$  while both lives are alive. The present value of the annuity payments is

expresses as a random variable, in terms of the joint future lifetime of  $x$  and  $y$ .

Derive and simplify as far as possible expressions for the expected present value and the variance of the present value of the annuity. **[6 marks]**

- (c). An office issues a policy on the lives of a woman aged 60 and her husband aged 64. Under this policy, level premiums are payable annually in advance for 20 years or until the first death of the couple, if earlier. On the first death of the couple, the survivor will receive an annuity of Ksh.10,000 per annum, payable weekly, beginning immediately on the first death. Calculate the annual premium if the office uses the basis given below:

Mortality: males:  $a(55)$  males ultimate  
              females:  $a(55)$  females ultimate  
Expenses: 20% of the first premium  
              5% of each premium after the first  
Interest: 6% per annum.

**[8 marks]**