DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2014/2015

FOURTH YEAR SEMESTER I EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MECHATRONICS ENGINEERING, ELECTRICAL AND ELECTRONIC ENGINEERING & TELECOMUNICATION AND INFORMATION ENGINEERING

SMA 2480 COMPLEX ANALYSIS

DATE: 18TH AUGUST 2014 **TIME: 8.30AM-10.30AM**

QUESTION ONE (30MKS)

a) Clearly distinguish between conformal and isogonal transformations (2mks)

b) State the Schwarz-Christoffel transformation and hence prove that for a closed polygon the sum of its exponents is equal to -2 (4mks)

c) Find the image of the circle $|Z| = $ a under the transformation $w = 5z$ (4mks)

d) Use Cauchy's integral formula to evaluate $\int_c \frac{z+4}{z^2+2z+5}dz$ where c is $|z+1-i|=2$ (5mks)

e) State the Cauchy's residue theorem and hence evaluate $\int_c \frac{e^z}{(z^2+®^2)^2}dz$ where c is $|z|=4$ (5mks)

f) Use the method of residues to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s-2)^2}\right\}$ (5mks)

g) Show that u is harmonic in some domain and find a harmonic conjugate v when $u = y^3 - 3x^2y$ (5mks)

QUESTION TWO (20MKS)

a) Evaluate $L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$ using residues method (7mks)

b) Evaluate $\int_{1+i}^{2+4i} z^2 dz$ along straight lines $(1+i)$ to $(2+i)$ and then to $(2+4i)$ (6mks)

c) Evaluate $\int_0^{2®} \frac{\cos 3\bar{}}{5-4\cos\bar{}}d\bar{}$ (7mks)

QUESTION THREE (20MKS)

a) Prove that a necessary condition for $w = f(z) = u(x,y) + iv(x,y)$ to be analytic is that the Cauchy Riemann's equations be satisfied in the region (7mks)

b) Given the transformation $w = f(z) = u + iv$ where w is analytic in a region $\Re$ then show that $J\left(\frac{u,v}{x,y}\right) = \left|f'(z)^2\right|$ (6mks)

c) Determine the region of the $w-$plane into which the region of the $z-$plane bounded by the straight lines $x=1, y=1$ and $x+y=1$ is mapped by the transformation $w=z^2$ (7mks)

QUESTION FOUR (20MKS)

a) Prove that $\cos(x+iy) = \cos x \cosh y - i \sin x \sinh y$ (4mks)

b) Verify whether the function $u(x,y) = \log\sqrt{x^2+y^2}$ satisfies the Laplace equation (i.e. harmonic) (4mks)

c) Show that $\int_0^{\infty} \frac{\cos mx}{x^2+1} dx = \frac{®}{2} e^{-m}, m > 0$ (6mks)

d) Using contour integration evaluate $\int_0^{2®} \frac{d^-}{5+3\sin^-}$ (6mks)

QUESTION FIVE (20MKS)

a) Find the bilinear transformation which the points $z=0, z=1$ and $z=\infty$ into the points $w=i, w=1$ and $w=-i$ (5mks)

b) Find the image of the region bounded by the lines $x=0, y=0$ and $x+y=1$ in the $z-$ plane by the mapping $w = ze^{i®/4}$ (7mks)

c) Evaluate $\int_0^{\infty} \frac{dx}{x^4+1}$ (8mks)