



**DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY**

**University Examinations 2013/2014**

**FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS AND THIRD  
YEAR FIRST SEMESTER BACHELOR OF SCIENCE IN MECHATRONIC  
ENGINEERING**

**SMA 2480: COMPLEX ANALYSIS**

**DATE: 15<sup>TH</sup> AUGUST 2013**

**TIME: 11.00-1.00 PM (2HRS)**

**INSTRUCTIONS: Answer questions ONE and any other TWO questions.**

**QUESTION ONE (30 MARKS)**

- a) Evaluate  $\frac{i^4+i^5}{2-i^5}$  **(4 marks)**
- b) Express  $\sqrt[3]{-1+i}$  in polar form and solve it and represent the roots graphically. **(5 marks)**
- c) Show that  $\frac{e^{z_1}}{e^{z_2}} = e^{z_1-z_2}$ . **(4 marks)**
- d) Given that  $z_1 = 3 + 4i$  and  $z_2 = 5 + 3i$  determine  $\overline{z_1 z_2}$ . **(3 marks)**
- e) Evaluate  $\oint_C \frac{z+1}{(z-2)^3(z^2+9)} dz$  around the square with vertices (0,0), (1,0), (1,1), (0,1) **(5 marks)**
- f) Evaluate  $\lim_{z \rightarrow i+1} \left[ \frac{z^2 - z + 1 - i}{z^2 - 2z + 2} \right]^{1/3}$  **(5 marks)**
- g) Determine if  $f(z) = \cos \bar{z}$  is analytic. If so find  $f'(z)$ . **(4 marks)**

**QUESTION TWO (20 MARKS)**

- a) Solve for real x and y given that  $\left(\frac{1+i}{1-i}\right)^6 + \frac{1}{x+iy} = 1+i$  **(7 marks)**

b) Evaluate  $\int_{3i}^{1-i} 4zdz$  along the straight line joining  $3i$  to  $1-i$ . (5 marks)

c) Find the residue of

$$f(z) = \frac{e^{2z}}{z-3}$$
 (4 marks)

d) Find the real numbers  $x$  and  $y$  such that  $3x + 2yi - xi + 5y = 7 + 5i$  (4 marks)

**QUESTION THREE (20 MARKS)**

a) Prove that

$$\int_0^{2\pi} \cos^{2n} \theta d\theta = \frac{1 \times 3 \times 5 \times \dots \times (2n-1) \times 2\pi}{2 \times 4 \times 6 \times \dots \times 2n}$$
 where  $n$  is a positive integer.

Hence evaluate  $\int_0^{2\pi} \cos^8 \theta d\theta$ . (12 marks)

b) Find the harmonic conjugate of the function  $f(z) = y^3 - 3x^2y$ . (8 marks)

**QUESTION FOUR (20 MARKS)**

a) Find the Laurent series about each singularity given that and classify the singularities stating the residue is found.

$$f(z) = \frac{z}{(z+1)(z+2)}$$
 (12 marks)

b) Evaluate

$$\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$$
 where  $C$  is  $|z| = 3$  (8 marks)

**QUESTION FIVE (20 MARKS)**

a) Name three types of singularities. (3 marks)

b) Evaluate

$$\int_{0.3}^{2.4} (2y + x^2)dx + (3x - y)dy$$

along;

i) The parabola

$$x = 2t, \quad y = t^2 + 3$$

**(7 marks)**

ii) Straight line from(0,3) to (2,3) and then from (2,3) to (2,4).

**(10 marks)**