W1-2-60-1-6

KIMATHI UNIVERSITY COLLEGE OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2010/2011

THIRD YEAR FIRST SEMESTER SUPPLEMENTARY EXAMINATION FOR BACHELOR OF SCIENCE IN MECHATRONICS AND TELECOMUNICATION AND INFORMATION ENGINEERING

SMA 2480: COMPLEX ANALYSIS

DATE: AUGUST 2011 **TIME: 2 HOURS**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY)

a) Distinguish between isolated singularity and a pole, give example in each case. [3 marks]

b) State Cauchy's Riemann equations. [2 marks]

c) Evaluate $\lim_{z\to 1}\left[\frac{z-1}{\sqrt{Z^2+3}-2}\right]$ [4 marks]

d) Describe the locus represented by $|z+2-3i|=5$ [4 marks]

e) Write the first three terms of the Taylor's series expansion of $\frac{1}{2z}$ about the point $z=2i$ [4 marks]

f) Evaluate $\int_{(0,3)}^{(2,4)} (x^2-y^2+2xy)dx+(3x-y)dy$ along the parabola $x = 2t$, $y = t^3 + 3$ [4 marks]

g) Show that $z=i$ is a removable discontinuity for $\frac{z^2+1}{z-i}$ [4 marks]

h) Prove that $\sin^3\theta = \frac{3\sin\theta}{4} - \frac{\sin 3\theta}{4}$ [5 marks]

QUESTION TWO

a) Simplify $\dfrac{1}{2-3i}$ and give its polar form [4 marks]

b) i) Define the harmonic function and harmonic conjugate function [4 marks]

ii) Show that the function $f(x,y)=x^4-6x^2y^2+y^4$ is harmonic everywhere and find its harmonic conjugate [6 marks]

c) Evaluate $\int_c \dfrac{e^{2z}}{(z+1)^2}dz$ where c is the circle |z| = 3 using the Cauchy's integral formula. Verify your answer by using the Residue's Theorem [6 marks]

QUESTION THREE

a) Find the residue of $f(z)=\dfrac{1}{(z)(z+2)^3}$ at the poles. [6 marks]

b) Evaluate $\int_c \dfrac{5z^2-3z+2}{(z-1)^3}dz$ where $|z|=3$ [6 marks]

c) Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for $1<|z|<3$. [8 marks]

QUESTION FOUR [20 MARKS]

a) Using the Cauchy Residue Theorem evaluate the integral $\int_0^{2\pi}\dfrac{d\theta}{3-2\cos\theta+\sin\theta}$ [8 marks]

b) If the real part of an analytic function is y^2-x^2-2y, determine

(i) the imaginary part

(ii) the function f(z)

(iii) f(z) for which f(0)=0 [7 marks]

c) Evaluate $(-1+i)^{1/3}$ [5 marks]

QUESTION FIVE [20 MARKS]

a) The complex potential $\Omega(z)$ is given by the relation $\Omega(z)=\Phi(x,y)+i\Psi(x,y)$. If the electrostatic potential $\Phi(x,y)=e^{-x}(x\sin y-y\cos y)$ and is harmonic, find the electrostatic flux $\Psi(x,y)$ [6 marks]

b) Prove that

(i) $\cos 5\theta=16\cos^5\theta-20\cos^3\theta+5\cos\theta$

(ii) $\sin 5\theta=5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+5\cos\theta$ [7 marks]

c) Evaluate $\int_{1-i}^{2+3i}(2z-5)dz$

(i) Along the straight line joining $1-i$ and $2-i$ [3 marks]

(ii) Along the straight line joining $1-i$ *and* $2+3i$ [4 marks]