DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY
UNIVERSITY EXAMINATION 2014/2015

SUPPLEMENTARY/SPECIAL EXAMINATION FOR
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC/MECHATRONIC/ TELECOMMUNICATION AND INFORMATION ENGINEERING

SMA 2480 COMPLEX ANALYSIS

DATE: 1ST JULY 2015 **TIME: 11.00AM-1.00PM**

QUESTION ONE (30MKS)

a) Prove that the function $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic (4mks)

b) Investigate the analyticity of the function $f(z) = |z^2|$ at any point z (5mks)

c) Use the Cauchy residue to evaluate $\oint_c \frac{2z^2+1}{(z-1)^3} dz$ (4mks)

d) If $f(z) = u + iv$ is analytic , find $f(z)$ if the real part is given by $u = \frac{\sin 2x}{\cos 2x + \cosh 2y}$ (5mks)

e) Under the transformation $w = iz + i$ show that the half plane $x > 0$ maps into the half-plane $v. > 1$ (6mks)

f) Evaluate $L^{-1}\left\{\frac{s}{(s+1)^3(s-1)^2}\right\}$ (6mks)

QUESTION TWO (20MKS)

a) Show that the function $x^2 - y^2 + 2y$ is harmonic in the w-plane under the transformation $z = w^2$ (6mks)

b) Evaluate $\int_0^{2\pi} \frac{\sin^2 \theta}{5 + 4\sin \vartheta}$ using contour integration (7mks)

c) Find a bilinear transformation which maps the points $1+i,-i,2-i$ at the $z-$plane into the points $0,1,i$ of the $w-$plane (7mks)

QUESTION THREE (20MKS)

a) Verify whether the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic and hence find the conjugate v (4mks)

b) Evaluate $L^{-1}\left(\frac{s^2}{(s^2+4)}\right)$ justifying all the steps (8mks)

c) Find the transformation which will map the interior of the infinite strip bounded by the lines $v=0$ and $v=\pi$ of the $w-$plane on to the upper half of the $z-$plane (8mks)

QUESTION FOUR (20MKS)

a) Use the Cauchy integral formula to evaluate $\int_c \frac{(1+z)e^z}{z^2(z+1)^2}dz$ where $c:(z+i)=\frac{1}{2}$ (6mks)

b) In aerodynamics and fluid dynamics, the functions ϕ and φ in $f(z)=\phi+i\varphi$ where $f(z)$ is analytic, are called the velocity potential and stream function respectively. If $\phi=x^2+4x-y^2+2y$,find φ and $f(z)$ (6mks)

c) Using contour integration, evaluate $\int_0^{\alpha} \frac{x^2dx}{(x^2+9)(x^2+4)}$ (8mks)

QUESTION FIVE (20MKS)

a) Find the map of the circle $|z|=\lambda$ by the transformation $w=\sqrt{2}(1+i)z$ (6mks)

b) Show that an arc in the $z-$plane through the point z_0 is magnified in the ratio $|f(z_0)|:1$ in the w-plane under the transformation $w=f(z)$, where $f(z)$ is an analytic function and $f'(z)\neq 0$ (7mks)

c) Evaluate $\int_0^{\alpha} \frac{dx}{x^4+1}$ (7mks)