COMPLEX ANALYSIS SMA 2480 TIE Y3S2

Instructions: Answer Question ONE and any other Two questions.

Question One

a) Solve for z: $3z\bar{z} - 4z = 3 - 6i$. (5 marks)

b) Let $z_1 = i$ and $z_2 = 1 - i$. Find $\frac{z_1}{z_2}$ in polar form. (4 marks)

c) Show that $f(z) = \bar{z}$ is non analytic anywhere. (5 marks)

d) Evaluate : $\int_{(2,0)}^{(0,2)} (z^2 + 3z)dz$. (5 marks)

e) Let $f(z) = u + iv$ be analytic. Show that $u = x^2 - y^3 + 3x - 2$ is a harmonic function and find its harmonic conjugate,v. (5 marks)

f) Find the complex potential due to a line of charge q per unit length perpendicular to the z plane at z=0 (6 marks)

Question Two

a) State and prove the cauchy's eimman equations. (10 marks)

b) Let the function f be defined by $f(z) = (\bar{z})^2$. Is $f(z)$ analytic anywhere in the z plane. Give your reasons. (6 marks)

c) Find the potential due to line charge q per unit length at and a line charge $-q$ per unit length at $z = \bar{z}_0$. (4 marks)

Question Three

a) Define bilinear transformation (2 marks)

b) Determine the region of the w-plane into which the region bounded by $x=1, y=1$ and $x+y=1$ is mapped by the transformation $w=z^2$. Show the region graphically. (8 marks)

c) Differentiate between conformal and isogonal mapping. (8 marks)

Question Four

a) State the residue theorem (2 marks)

b) Evaluate: $\frac{1}{2\pi i}\oint_c \frac{e^{zt}}{z^2(z^2+2z+2)}dz$ around a circle: $|z|=3$. (10 marks)

c) Evaluate (i) $\lim_{z \to i} \frac{z^{10}+1}{z^6+1}$

(ii) $\frac{dw}{dz}$ if $w^3 - 3z^2w + 4\ln z = 0$ (8 marks)

Question Five

a) (i) State the Cauchy's integral formulae. (2 marks)

(ii) Evaluate $\int \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}dz$ by Cauchy's integral formula (8 marks)

b) Evaluate $\int_{(0,3)}^{(2,4)} (2y+x^2)dx + (3x-y)dy$ along

(i) The parabola $x = 2t, y = t^2 + 3$ (4 marks)

(ii) Straight lines from (0,3) to (2,30 and then from (2,3) to (2,4) (3marks)

(iii) A straight line from (0,3) to (2,4) (3 marks)