DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2020/2021

SPECIAL AND SUPPLEMENTARY EXAMINATION

FIRST SEMESTER FOURTH YEAR DEGREE IN BACHELOR OF SCIENCE IN

ACTUARIAL SCIENCE

SAS 4190: FINANCIAL ECONOMICS

Date: ?? 2020   Time: 2hrs

Instructions: Attempt Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) i) Outline the three forms of the Efficient Markets Hypothesis (EMH).   (4 marks)

ii) Discuss the following two scenarios in the light of the EMH: (5 marks)

     Scenario 1: Company A's share price falls suddenly, immediately after news of an

                           earthquake in the capital city of one of its major markets.

     Scenario 2: Company B's share price falls suddenly, when a long-awaited and publicly

                          negotiated merger is completed.

b) Assume CAPM holds in the following market. The market consists of 3 risky securities
    X, Y, Z. Standard deviation of annual returns of these three securities is 45%, 30% and
    15% respectively. The market capitalization of these securities is in the same proportion
    as their standard deviation.   The risk-free rate of interest is 10%.   The correlation
    coefficients between the returns are given the table below.

Pair	Correlation
X, Y	0.75
Y, Z	0.20
X, Z	-0.10

Derive the covariance matrix for the returns on X, Y and Z.   (8 matrix)

c) Consider a European call option and a European put option on a non-dividend-paying stock. You are given:

i) The current price of the stock is 60.

ii) The call option currently sells for 0.15 more than the put option.

iii) Both the call option and put option will expire in 4 years.

iv) Both the call option and put option have a strike price of 70

Calculate the continuously compounded risk-free interest rate.   (7 marks)

d) Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
$40	$11	$3
$50	$6	$8
$55	$3	$11

All six options have the same expiration date.

After reviewing the information above, John tells Mary and Peter that no arbitrage opportunities can arise from these prices.

Mary disagrees with John. She argues that one could use the following portfolio to obtain arbitrage profit: Long one call option with strike price 40; short three call options with strike price 50; lend $1; and long some calls with strike price 55.

Peter also disagrees with John. He claims that the following portfolio, which is different from Mary's, can produce arbitrage profit: Long 2 calls and short 2 puts with strike price 55;

long 1 call and short 1 put with strike price 40; lend $2; and short some calls and long the same number of puts with strike price 50. Which is the correct scenario ?   (6 marks)

QUESTION TWO (20 MARKS)

    a)   For a two-period binomial model, you are given:

    i)   Each period is one year.

    ii) The current price for a nondividend-paying stock is 20.

    iii) $U=1.2840$, where u is one plus the rate of capital gain on the stock per period if   the

$$r_{uu} = 18\%$$

$$r_u = 15\%$$

$$r_{ud} = 12\%$$

$$r_0 = 12\%$$

$$r_{du} = 12\%$$

$$r_d = 9\%$$

$$r_{dd} = 6\%$$
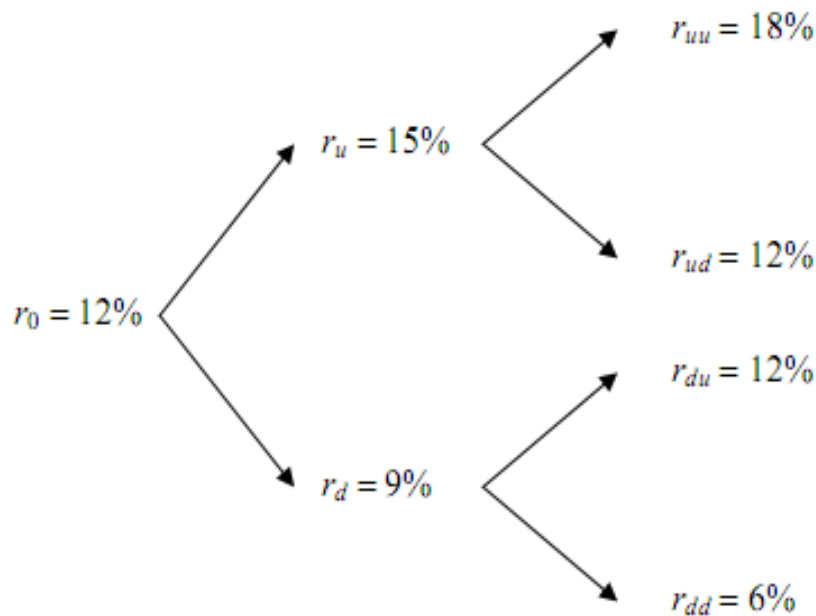
        stock price goes up

    iv) $d=0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock

        price goes down.

    v) The continuously compounded risk-free interest rate is 5%.

 Calculate the price of an American call option on the stock with a strike price of 22.   (6 marks)

    b)   You are given the following three-period interest rate tree. Each period is one year. The

       risk-neutral probability of each up-move is 70%. The interest rates are continuously

       compounded rates

Consider a European put option that expires in 2 years, giving you the right to sell a one-year zero-coupon bond for 0.90. This zero-coupon bond pays 1 at maturity. Determine the price of

the put option. (6 marks)

c) Consider a two-period binomial model for a non-dividend paying stock whose current price is 0100s

Assume that:

- over each of the next six-month periods, the stock price can either move up by a factor 1.2u or down by a factor 0.8d.
- the continuously compounded risk-free rate is 6% r per period

i) Prove that there is no arbitrage in the market (4 marks)

ii) Construct the binomial tree for the model (4 marks)

QUESTION THREE (20 MARKS)

a) The continuously compounded risk-free rate of interest is r, and a stock, with maturity T, pays dividends continuously at rate q.

i) Determine the forward price at time 0 for a forward contract on the stock

(5 marks)

ii) Show that there exists a portfolio that earns the risk free rate r, containing:

- The stock
- A European call option on the stock

- and a European put option on the stock

(4 marks)

b) A non-dividend-paying stock has a current price of 300p. Over each of the next two three-month periods its price will either go up by 30p or down by 30p. Price movements for each period are independent of each other. An investment in a cash account returns 2% per quarter. A European call option on the stock pays out in six months based on a strike price of 290p. The price of the stock is to be modelled using a binomial tree approach with three-month time steps.

i) Calculate the value of the call option today using a risk-neutral pricing approach.

(4 marks)

Assume that the real world probability of the stock price moving up in each of the next three month periods is 0.7

ii)   Calculate the values of the state price deflator after six months and the value of
        the call option today   (4 marks)

iii)   Compare this to your answer to part (i)   (3 marks)

QUESTION FOUR (20 MARKS)

a) The current gross value of XYZ plc is $180m. The continuously compounded risk- free
    interest rate is 2% p.a. and the continuously compounded credit spread on the bond is
    4.5% per annum.

i) Calculate the price of the bond today.   (4marks)

ii) Estimate to the nearest 1% the implied volatility of the value of XYZ.   (3 marks)

iii) Determine the implied risk-neutral probability of default.   (4 marks)

b) Assume CAPM holds in the following market. The market consists of 3 risky securities
    X, Y, Z. Standard deviation of annual returns of these three securities is 45%, 30% and
    15% respectively. The market capitalization of these securities is in the same proportion
    as their standard deviation.

    The risk-free rate of interest is 10%.

    The correlation coefficients between the returns are given the table below.

Pair	Correlation
X, Y	0.75
Y, Z	0.20
X, Z	-0.10

        Derive the covariance matrix for the returns on X, Y and Z.

        Calculate the expected returns of X, Y and Z if the expected return on the market
portfolio is 30%. Discuss the main problems involved in estimating parameters for asset pricing
models.   (4 marks)

c) Assume an investor has initial wealth of 1 unit and his utility function has the
    Following form

$$U(w) = \frac{w^{\gamma} - 1}{\gamma} \quad \text{for } w > 0$$

i) For what values of γ will the investor be considered as risk averse?   (5 marks)

QUESTION FIVE (20 MARKS)

a) The expected utility function for a young actuarial student, who wishes to construct a portfolio consisting of a risk-free and a risky asset, is

$$E(U) = r_a - \frac{1}{2}\sigma_a^2$$

Where r_a and σ_a are the mean and standard deviation of the portfolio rates of return. The risk-free asset has an expected rate of return of 4% p.a. The risky asset has an expected rate of return of 12% p.a. and variance of 10%% p.a.

Determine the portfolio that will maximize the investor's expected utility.   (5 marks)

b) Returns for a security is are distributed as $R \sim Exp(\lambda)$. Derive expressions for variance and downside semi variance. Evaluate the derived expressions for $\lambda = \frac{1}{2}$.   (7 marks)

c) Derive the equation of the efficient frontier in expected return-standard deviation space for a portfolio consisting of two stocks with standard deviations σ_1 and σ_2 and are perfectly correlated.   (5 marks)

d) Name three types of credit risk models. Which type is the Merton model?   (3 marks)