THIRD YEAR FIRST SEMESTER & FOUTH YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY IN BUILDING & CONSTRUCTION

TBD 2303: ENGINEERING MATHEMATICS I

DATE: AUGUST 2021 **TIME: 2 HOURS**

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

QUESTION ONE (30 MARKS)

a) Find the domain of the function and represent it on a graph.

$f(x,y)=\sqrt{16-x^2-4y^2}$ (4 marks)

b) Evaluate

i) $\lim_{x\to\infty}\frac{\sqrt[3]{5+8x^6}}{7-5x^2}$ (5 marks)

ii) $\lim_{x\to 0}\frac{e^{2x}-e^{-2x}}{\sin x}$ (3 marks)

c) If the function $f(x)=\begin{cases}\frac{4-x^2}{x-2} & x\neq 2\\ P & x=2\end{cases}$

is continuous, find the value of P (4 Marks)

d) Show that if $u=\ln\sqrt{x^2+y^2}$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (5 marks)

e) Find the maclaulin series for the function $f(x)=e^{3x}\sin 2x$ (5 marks)

f) Given the function $z = xy^2 - 8x$ and that $x = 1$, $y = -3$, $\Delta x = -0.01$ *and* $\Delta y = 0.02$

a) Determine value of Δz *and* dz (4 marks)

QUESTION TWO (20 MARKS)

a) i) State the mean value theorem of differential calculus.

ii) Find the mean value of the function $f(x) = 2x^2 - 7x + 10$ on the interval [2, 5]

(5 marks)

b) Find the value of $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x \partial y}$ given $z = \frac{\sin x^3 y}{x^3}$

where z is a function defined by two variables x *and* y (5 marks)

c) Find the extreme values of the function $f(x, y) = x^2 + 8xy + 7y^2 = 0$ (5 marks)

d) Use Maclaurin's series approximation to express $\log_e(1+x)$ as a polynomial of the fourth degree. Use your result to evaluate $\log_e 1.2$ (5 marks)

QUESTION THREE (20 MARKS)

a) State Rolle's theorem

Hence justify Rolle's Theorem for the function $f(x) = x^3 - 3x^2 + x + 1$ on the interval $\left[1, 1+\sqrt{2}\right]$ (5 marks)

b) Find the value of $\frac{\partial^2 f}{\partial x \partial y}$ for the function $f(x, y) = y + x^2 y^{-2} + 4y^3 e^{-4x} - \ln\left(y^2 + x\right)$

(4 Marks)

c) A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is a minimum? (6 marks)

d) By considering the paths along $x = 0$, $y = 0$ *and* $kx^2 = y$ show that the function $f\left(x.y\right) = \dfrac{x^4 - y^2}{x^4 + y^2}$ has no limit as $\left(x, y\right) \to \left(0, 0\right)$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the extrema of the function $f\left(x, y\right) = 4x - 2y$ subject to constraint $x^2 + y^2 = 1$ using the Lagrange's multiplier. (4 marks)

b) Find values of A and B for which the function is continuous for all values of x

$$f\left(x\right) = \begin{cases} Ax - B & x < -1 \\ 2x^2 + 3Ax + B & -1 \leq x \leq 1 \\ 4 & x > 1 \end{cases}$$ (5 marks)

c) Given that $f\left(x, y\right) = \sin xy + xe^y$ show that

$$\left.\frac{\partial f}{\partial x}\right|_{(0,3)} - \left.\frac{\partial f}{\partial y}\right|_{(2,0)} = e^3 - 1$$ (4 marks)

d) Use Taylors approximation to express $\cos\left(\frac{®}{3} + h\right)$ in ascending powers of h up to h^4

Taking $\sqrt{3} = 1.7321$ and 5.5^0 *as* 0.09599 radians, find the value of $\sin 54.5$ (7 marks)

QUESTION FIVE (20 MARKS)

a) State Rolle's theorem, hence verify Rolle's theorem for
$f\left(x\right) = 1 + \cos 2x$ in the interval $\left[0, ®\right]$ (4 marks)

e) Find Taylor series expansion of the integral at $x_0 = 0$

$$\int x^3 \ln 4x$$ (5 marks)

b) Determine the critical points of the function $f(x, y) = x^2 + 3xy + y^2$ (5 marks)

c) Use limits to find all the asymptotes

$$f(x) = \frac{x^3 - x}{x^2 - 6x + 5}$$ (6 marks)